Causal analysis of drop deformation and breakup in isotropic turbulence

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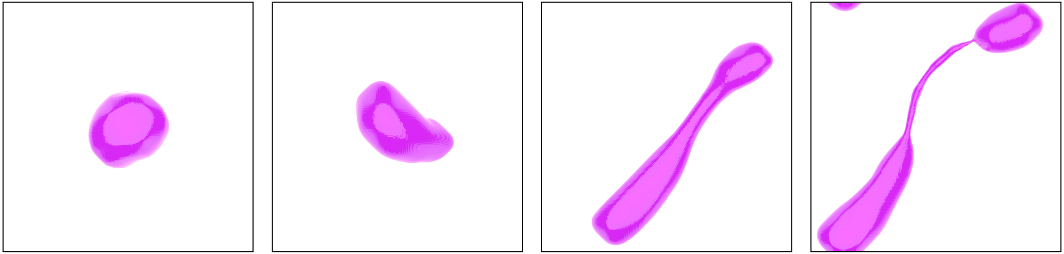
Ianto Cannon, Daniel Morón, Alberto Vela-Martín and Marc Avila



Motivation: Emulsions

- Emulsions are droplets of one liquid in another.
- In these systems, one liquid is broken into smaller and smaller drops by **turbulent stresses**.
- Drop breakup is a competition between inertia and surface tension (Hinze 1955):

$$\mathrm{We} = \frac{\rho u_d^2 d}{\sigma},$$

- ρ: fluid density
- σ: surface tension
- d: drop diameter
- u_d: velocity fluctuations
- $t_d = d/u_d$: eddy turnover time.

Alioli is an example of an emulsion

Open Questions

Despite decades of research, there are open questions (Ni 2024):

1. What are the timescales of drop deformation?
2. What are the timescales of the breakup process?
3. Which features of the turbulent flow correlate with breakup?.

Alioli is an example of an emulsion

Direct numerical simulation of the Cahn-Hilliard Navier-Stokes equations

Hundreds of independent simulations

Spatial resolution:

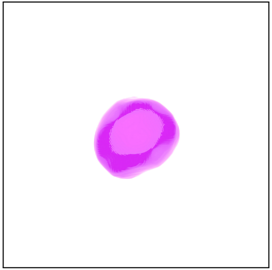
- Triperiodic domain L^3
- GPU-accelerated pseudospectral method
- 256^3 grid points
- $\mathrm{Re}_\lambda = 58$
- Drop diameter: $d = 0.3L$

Temporal resolution:

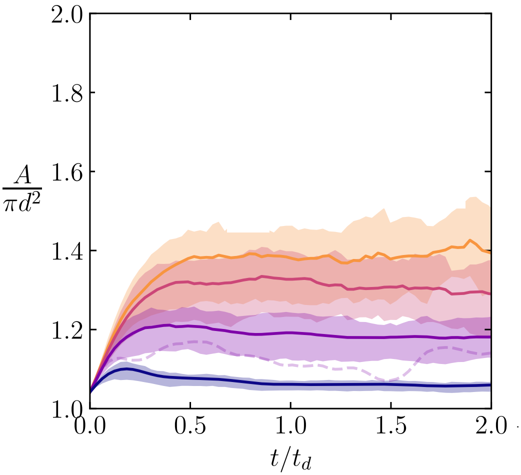
- Save every $0.03t_d$
- Max duration: $6t_d$
- Stop at breakup detection

youtu.be/AEsZjPGtYoA?si=_45uvXxNGmDuncek.

We	N_0	Colour
0.726	201	
1.82	200	
2.90	499	
3.63	599	



Equilibrium drop area



- Drops reach a statistically steady area within one eddy turnover time
- The fit:

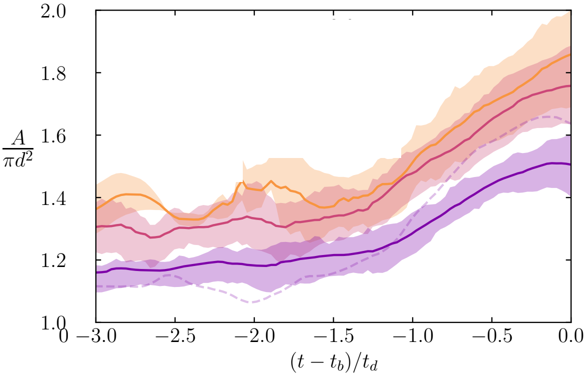
$$\frac{\langle A \rangle}{\pi d^2} = 0.111\mathrm{We} + 0.98$$

- Can be used to calculate equilibration energy:

$$\frac{\sigma \langle A \rangle - \sigma \pi d^2}{\frac{1}{2}\rho u_d^2 d^3} = 2\pi \times 0.111 = 69.7\%$$

- Independent of **surface tension**
- Depends only on the kinetic energy of the **turbulent eddies**
- Analogous to **equipartition theorem**.

Drop area at breakup



- Drops area is maximum at breakup
- The fit:

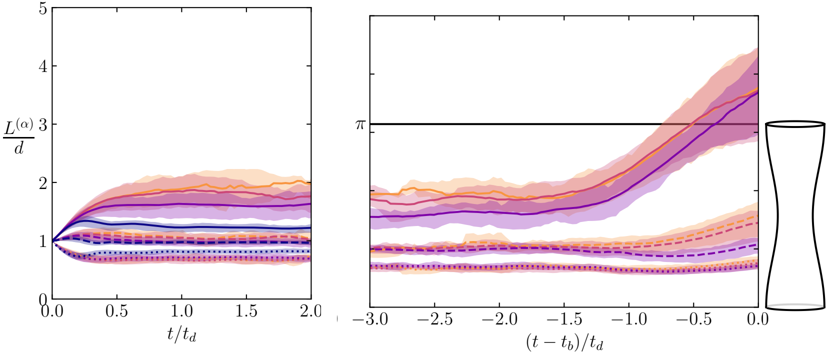
$$\frac{A_b}{\pi d^2} = 0.198\mathrm{We} + 1.16$$

- Gives the energy increase during breakup:

$$\frac{\sigma A_b - \sigma \langle A \rangle}{\frac{1}{2}\rho u_d^2 d^3} = 2\pi \left(0.086 + \frac{0.175}{\mathrm{We}}\right)$$

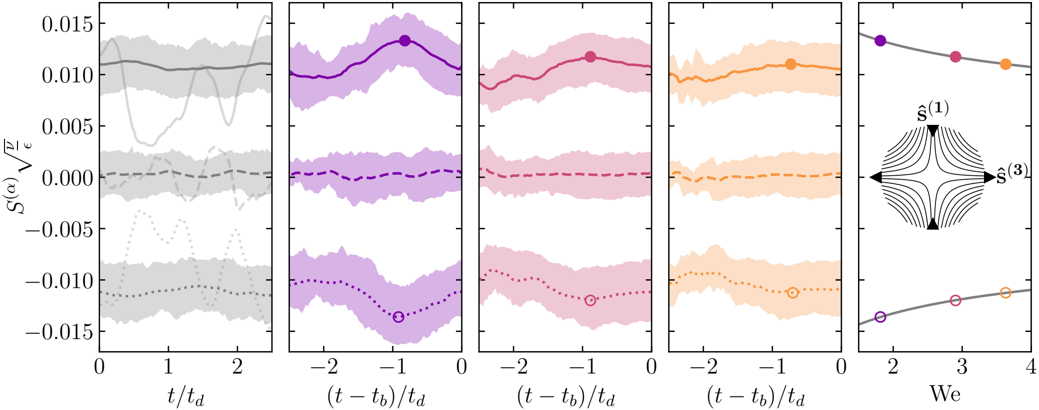
- For small We, second term dominates
- Surface tension creates an energy barrier to fragmentation.

Drop principal axes



- Axes lengths from the moment of inertia tensor $L^{(1)} < L^{(2)} < L^{(3)}$
- Aspect ratio $\to 1$ as $\mathrm{We} \to 0$
- πd = minimum unstable wavelength for Rayleigh-Plateau instability (Rayleigh 1878).

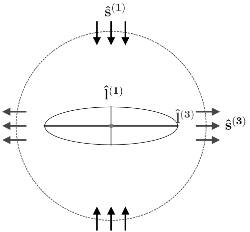
Turbulent strain rate before breakup

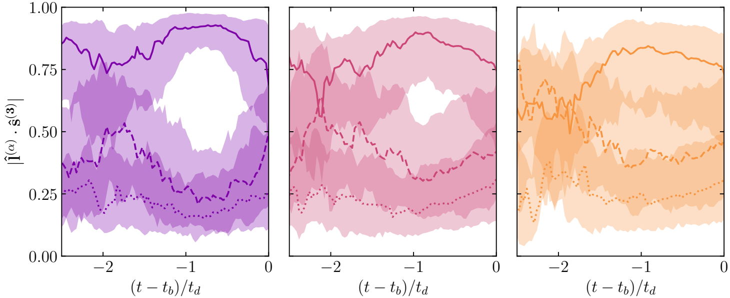


- Strain rate averaged in sphere of diameter $2d$, eigenvalues: $S^{(1)} \leq S^{(2)} \leq S^{(3)}$
- Strain rate peaks $\sim 1.0 t_d$ before breakup: $S^{(3)}_{\mathrm{max}} \propto \mathrm{We}^{-0.27}$.

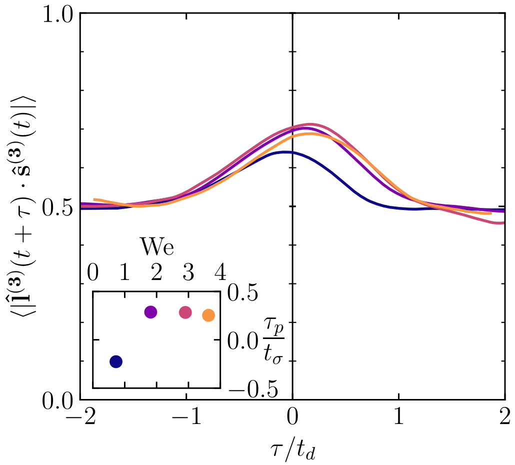
Alignment with extensional strain

- Measure alignment $\cos\theta = |\hat{\mathbf{l}}^{(\alpha)} \cdot \hat{\mathbf{s}}^{(3)}|$
- $|\hat{\mathbf{l}}^{(\alpha)} \cdot \hat{\mathbf{s}}^{(3)}| = 1$: drop axis **parallel** to strain direction
- $|\hat{\mathbf{l}}^{(\alpha)} \cdot \hat{\mathbf{s}}^{(3)}| = 0$: drop axis **perpendicular** to strain direction
- $|\hat{\mathbf{l}}^{(\alpha)} \cdot \hat{\mathbf{s}}^{(3)}| = 0.5$: drop axis **uncorrelated** with strain direction
- Long axis $\hat{\mathbf{l}}^{(3)}$ aligns with extensional direction $\hat{\mathbf{s}}^{(3)}$
- Peak alignment $\sim 1.0 t_d$ before breakup.



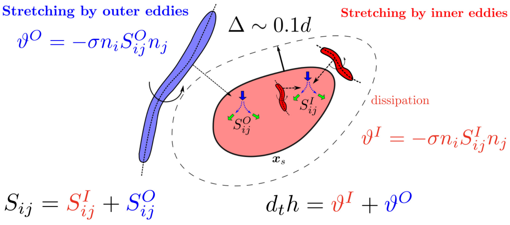


Temporal correlation of strain and deformation

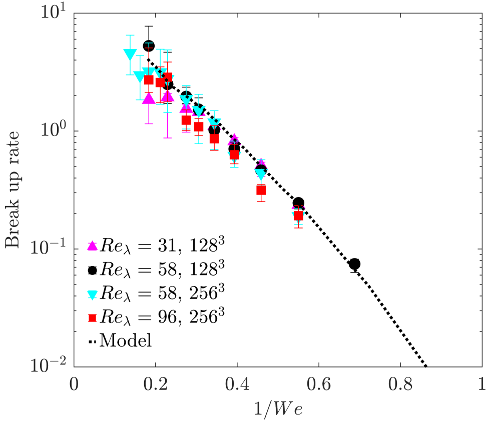


- Measure alignment of the drop axis $\hat{\mathbf{l}}^{(3)}$ with the strain direction $\hat{\mathbf{s}}^{(3)}$ with a time delay τ.
- Drop response time is $\tau_p \approx 0.25 t_\sigma$ for $\mathrm{We} > 1$.

One dimensional model of drop breakup



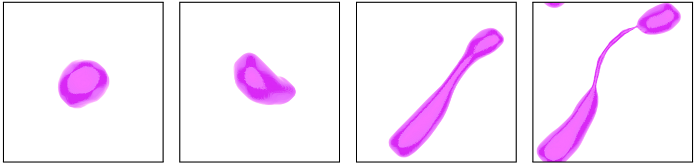
- We measure the probability distributions of **inner stretching**, and **outer stretching** (Vela-Martín and Avila 2021)
- We make a stochastic model of the drop area which accurately predicts the breakup rate (Uhlenbeck and Ornstein 1930).



Conclusions

- Performed hundreds of DNS across $\mathrm{We} = 0.726$ to 3.63
- Identified three timescales: $t_0 \approx t_d$ (equilibration), t_d (forcing), t_σ (response)
- Energy equipartition during equilibration: 69.7% of eddy kinetic energy
- Breakup threshold: $L^{(3)} > \pi d$ (Rayleigh-Plateau instability)
- Drop response lag: $\tau_p \approx 0.25 t_\sigma$ for $\mathrm{We} > 1$
- Results enable a reduced-order model of drop breakup

Thank you!



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