# Drops and surfactant in turbulent flows

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We use direct numerical simulations to investigate the dynamics of clean (surfactant-free) and surfactant-laden droplets in homogenous isotropic turbulence at moderate Taylor Reynolds number. We show that surfactant effects are well approximated using a lower, averaged value of surface tension, allowing us to extend the framework developed by Kolmogorov (1949) and Hinze (1955) for surfactant-free bubbles to surfactant-laden droplets. The Kolmogorov-Hinze scale, $d_H$, is indeed a pivotal length scale in the droplets' dynamics, separating the coalescence-dominated and the breakage-dominated regimes in the droplet size distribution. We see that droplets smaller than $d_H$ have spheroid-like shapes, whereas larger droplets have long convoluted filamentous shapes with diameters equal to the $d_H$. As a result, droplets smaller than $d_H$ have areas that scale as $d^2$, while larger droplets have areas that scale as $d^3$, where $d$ is the droplet equivalent diameter.

### Numerical method

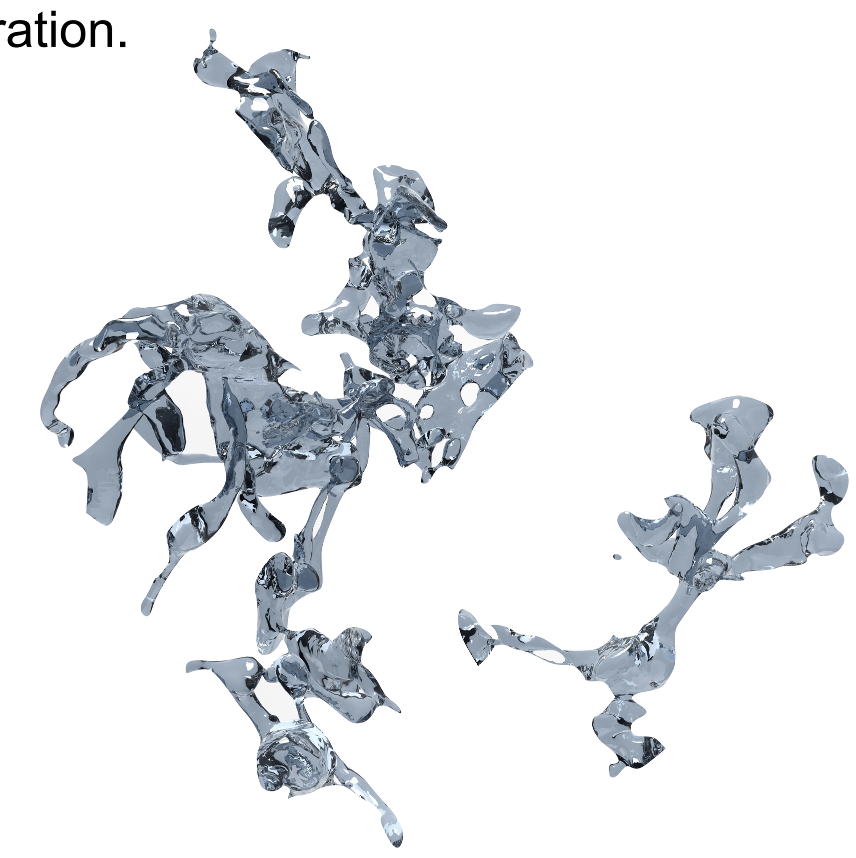
The dynamics of the turbulent flow are computed via the Navier-Stokes and continuity equations; turbulence is maintained via the forcing term $\mathbf{f_s}$ and surface tension forces are accounted for in the $\nabla \cdot (\tau_c f_\sigma)$ term. The volume of fluid variable, $\phi$, characterizes the local concentration of the droplet phase. We consider a soluble surfactant, with $\psi$ being its concentration; the value of the surface tension depends on the local surfactant concentration.

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla \cdot \left[ \mu \left( \nabla \mathbf{u} + \nabla \mathbf{u}^T \right) \right] + \nabla \cdot (\tau_c f_\sigma) + \mathbf{f_s}$$ Navier-Stokes equation

$$\nabla \cdot \mathbf{u} = 0$$ Continuity equation

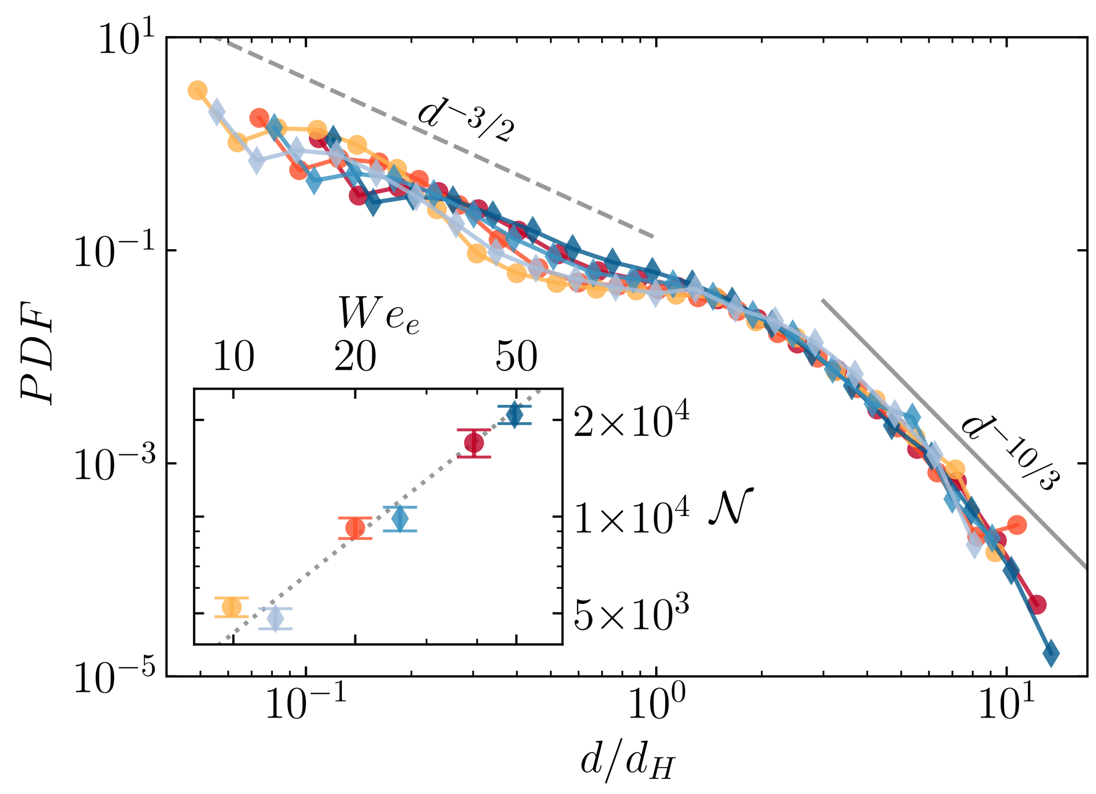
$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\mathbf{u} H) = 0$$ Volume of fluid transport equation

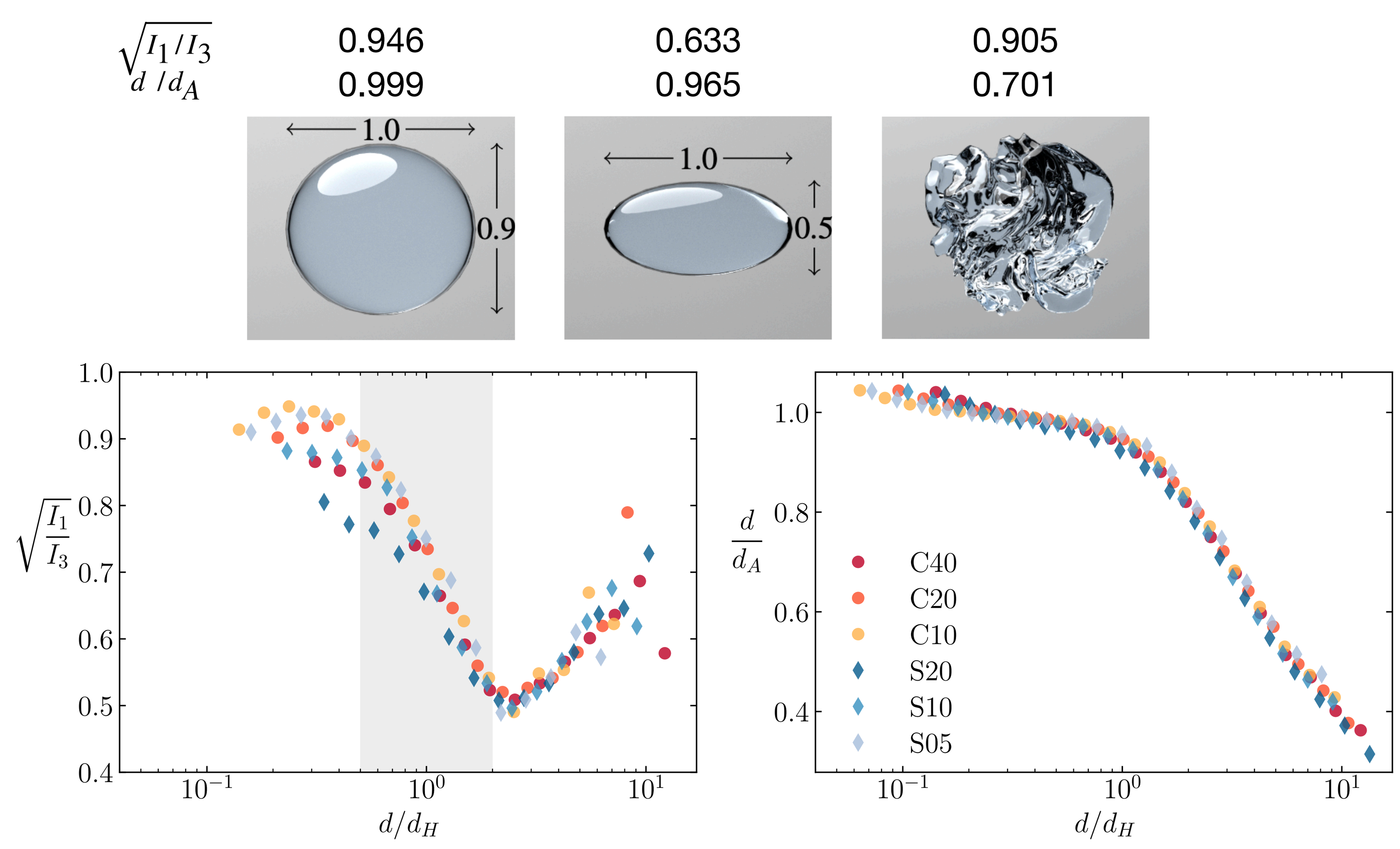
$$\frac{\partial \psi}{\partial t} + \mathbf{u} \cdot \nabla \psi = \nabla \cdot (M_\psi \nabla \mu_\psi)$$ Surfactant transport equation



The numerical simulations are performed at a moderate Taylor Reynolds number, $Re_\lambda \approx 180$, with different values of the reference Weber number, ratio of inertial contributions over surface tension forces. We investigate a total of seven cases, a reference single phase case (SP •), three surfactant-free cases (C10 •, C20 •, C40 •) and three surfactant-laden cases (S05 ♦, S10 ♦, S20 ♦). The number in the name of each case is the reference Weber number.

**Droplet size distribution** (DSD): the DSD for all cases collapse on a single line when normalized by the Kolmogorov-Hinze scale. The Kolmogorov-Hinze scale separates the coalescence-dominated regime ($d/d_H < 1$) and the breakage-dominated regime ($d/d_H > 1$). Data from our numerical simulations for both surfactant-free and surfactant-laden cases follow the power-law scalings identified for the two regimes. **Droplet count** (inset): the number of droplet at statistical steady state follows is a linear function of the effective Weber number, $We_e$. The effective Weber number is computed by rescaling the reference Weber number by the average surface tension value at the interface.

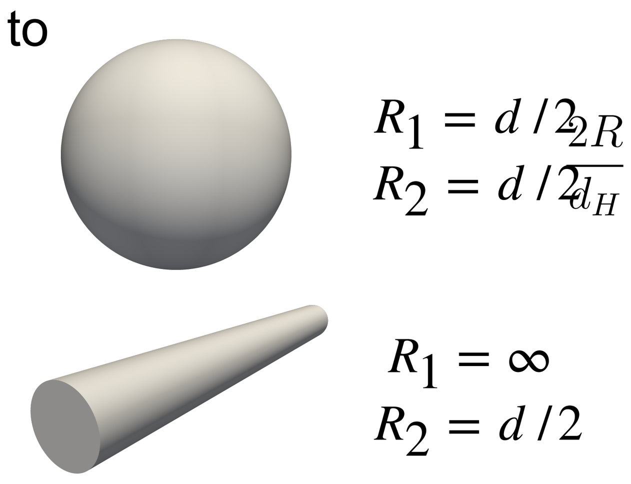


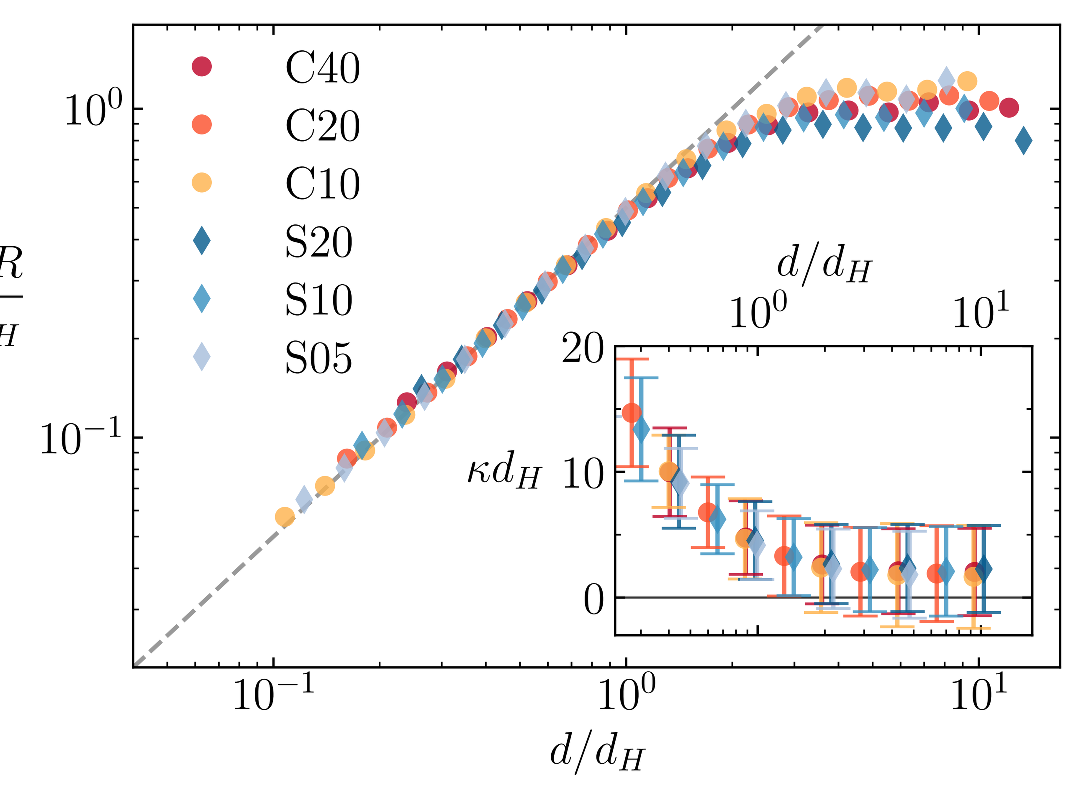



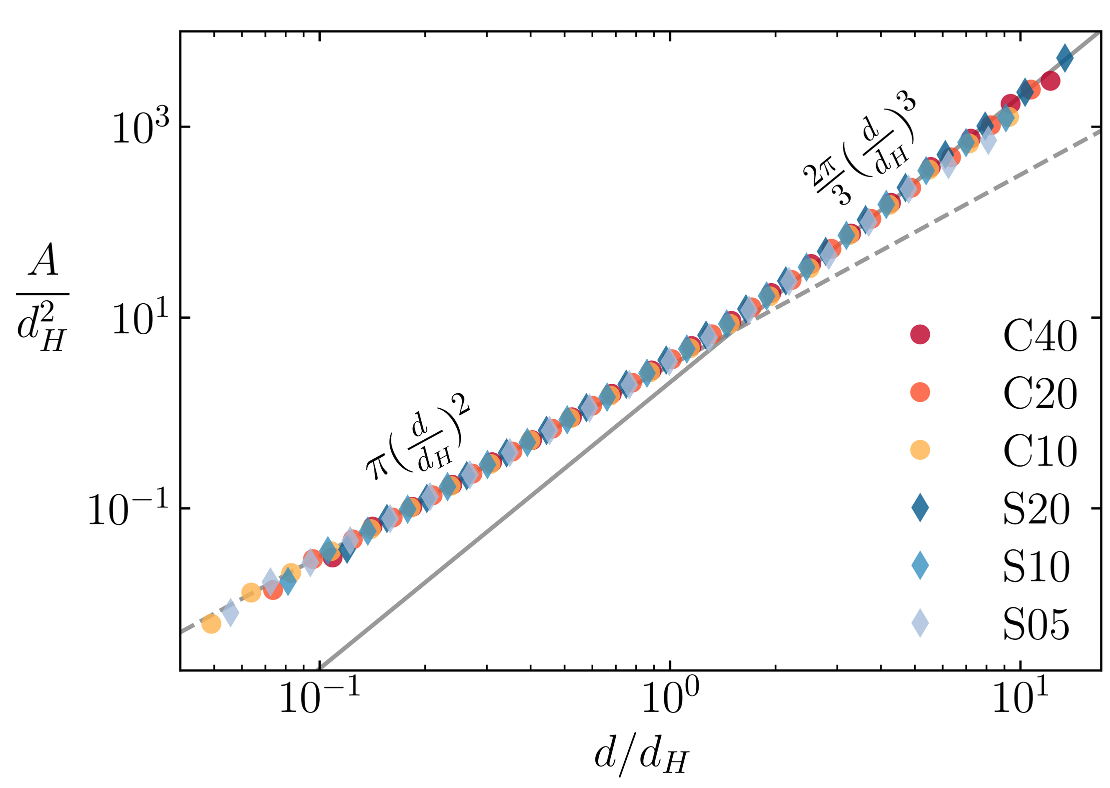

**Deformation parameters** (top panel): the aspect ratio $\sqrt{I_1/I_3}$ is highly sensitive to large scale deformation, and the sphericity $d/d_A$ is highly sensitive to local interfacial perturbations. **Aspect ratio** (bottom left panel): droplets smaller than $d_H$ have spheroidal shape. A strong deformation is observed for droplets of size about $d_H$; a reduction in the deformation is reported above $d_H$. **Sphericity** (bottom right panel): droplets smaller than $d_H$ have a regular, spheroid-like shape, while larger droplets have a more convoluted shape (filaments or bulgy droplets).

**Radius of curvature**: the mean radius of curvature for droplets smaller than $d_H$ grows linearly with their characteristic size, whereas for larger droplets it is equal to $d_H/2$. Thus large droplets have convoluted filamentous shapes.

$$R = \frac{R_2}{1 + R_2/R_1}$$




**Interfacial area**: the interfacial area of droplets smaller than $d_H$ grows as the square of their characteristic size $d$, whereas for droplets larger than $d_H$ we report a cubic growth with $d$.

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**References**
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