Spatio-temporal energy spectra of drops in turbulence

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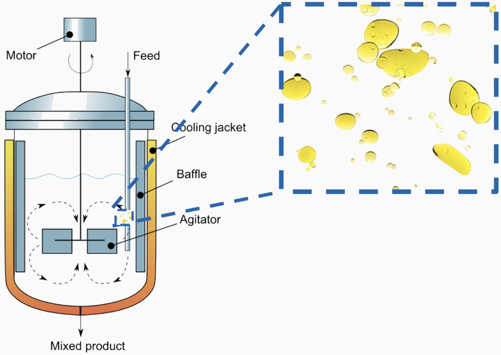
Drop breakup in turbulence

- Occurs in emulsification of foods, paints, and pharmaceuticals [Mathijssen et al., *Reviews of Modern Physics*, 2023]
- Competition between inertia and surface tension [Hinze, *AIChE J.*, 1955]:

$$\mathrm{We} = \frac{\rho u_d^2 d}{\sigma}$$

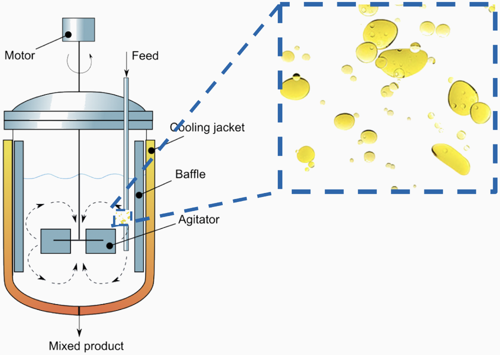
- ρ: fluid density
- d: drop diameter
- u_d: velocity fluctuations
- σ: surface tension
- $t_d = d/u_d$: eddy turnover time.



Open Questions

Despite decades of research, there are open questions [Ni, *Annu. Rev. Fluid Mech.*, 2024]:

1. What size turbulent eddies cause drop deformation?
2. What are the timescales of the breakup process?
3. Can we make a mechanistic model of drop breakup?



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Cahn–Hilliard–Navier–Stokes equations

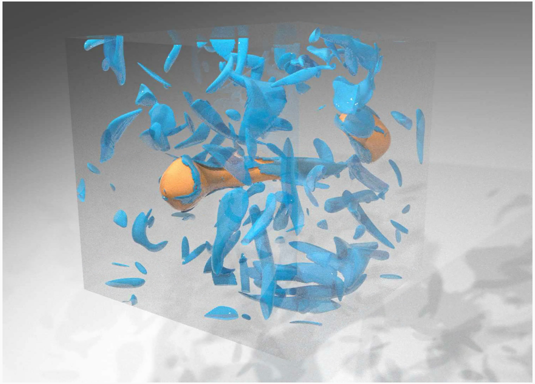
Uniform density ρ and viscosity ν:

$$\rho\,\partial_t u_i + \rho\,u_j\partial_j u_i = \rho\nu\,\partial_{jj}u_i - \partial_i p + \zeta u_i + f_i$$
$$\partial_i u_i = 0$$
$$\partial_t c + u_j\partial_j c = m\,\partial_{ii}\phi$$

with interfacial force $f_i = -c\,\partial_i\phi$ and chemical potential

$$\phi = \beta(c^2 - 1)c - \alpha\,\partial_{ii}c.$$

Phase-field method (unlike VOF [Paulista et al., 2007] or front-tracking [Unverdi et al., *J. Comp. Phys.*, 1992]): interfacial energy is built into the free energy functional.



Direct numerical simulation of the Cahn-Hilliard Navier-Stokes equations

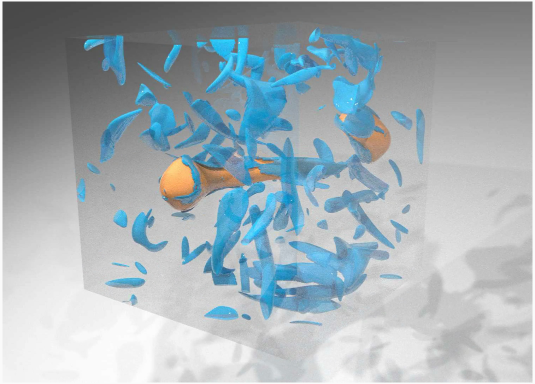
Hundreds of independent phase-field simulations [Magaletti et al., *J. Fluid Mech.*, 2013]

Spatial resolution:

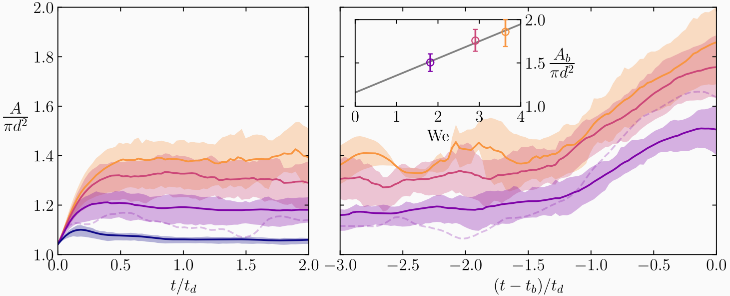
- Triperiodic domain L^3
- GPU-accelerated pseudospectral method
- 256^3 grid points
- $\mathrm{Re}_\lambda = 58$
- Drop diameter: $d = 0.3L$

Temporal resolution:

- Save every $0.03t_d$
- Stop at breakup detection

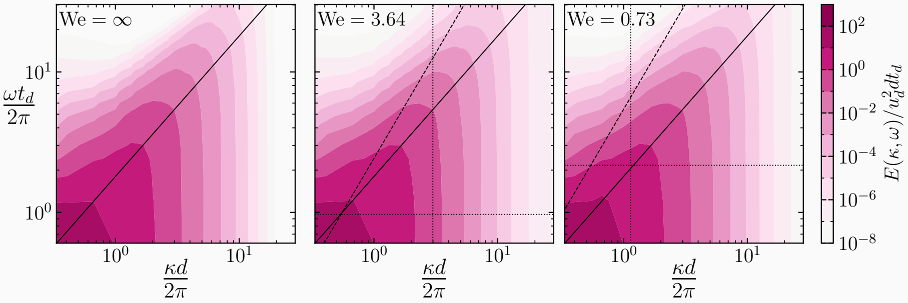


Mean drop area



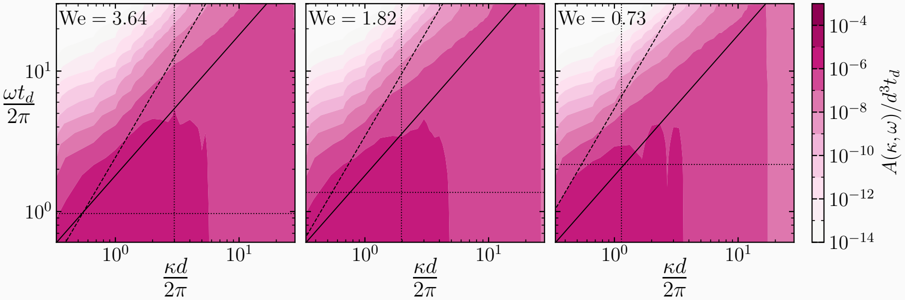
- Drops reach a statistically steady area within one eddy turnover time
- The equilibrium area increases with We
- Breakup occurs area increases by $0.27\pi d^2$
- We exclude $1t_d$ from the beginning and end of each signal
- We make ϕ and $\mathbf{u}$ periodic in time by multiplying with a Hann window $H(t) = \sin^2 \frac{\pi(t-t_d)}{3t_d}$.
- We make a 4D Fourier transform time and space.
- Isotropy $\Rightarrow$ integrate over spherical shells in κ-space.

Turbulent kinetic energy $E(\kappa,\omega) = \frac{1}{2}\langle \hat{u}_i^* \, \hat{u}_i \rangle$



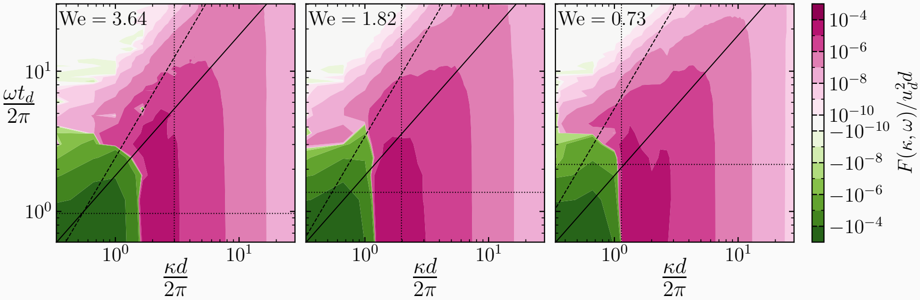
- Energy concentrated at large, slow scales.
- Ridge near $\omega = u'\kappa$ — eddies are advected by the large scale flow
- Drops produce a modest increase at small scales and short timescales.
- Dotted: Hinze scale $2\pi/d_H$ and capillary frequency $2\pi/t_\sigma$.
- Dashed: capillary wave $\omega^2 = \sigma\rho^{-1}\kappa^3$.

Interfacial area spectrum $A(\kappa,\omega) = \left\langle \kappa_i \kappa_i \, \hat{\phi} \, \hat{\phi}^* \right\rangle$



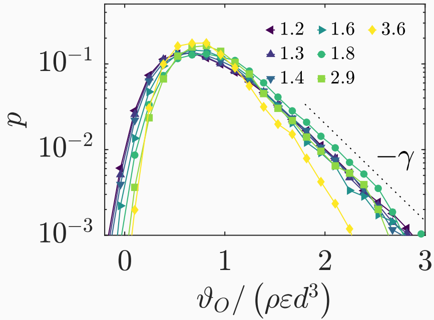
- Energy at low frequencies, peak at the drop-diameter length scale.
- Larger fraction stored at *small* length scales than in $E(\kappa,\omega)$.
- Clear ridge near $\omega = u'\kappa$ — advective transport of interfacial structures.

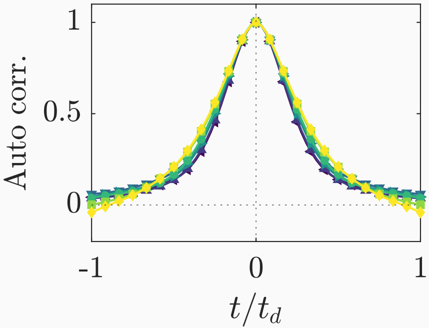
Interfacial forcing $F(\kappa,\omega) = \langle \hat{u}_i^* \hat{f}_i \rangle$



- Negative at large scales, long times: the drop *absorbs* energy.
- Positive at small scales, short times: the drop *returns* energy.
- Crossover near drop diameter d and inertial time t_d.
- As $\mathrm{We} \to 0$, crossover $\to (d, t_d)$ — rigid-sphere limit.

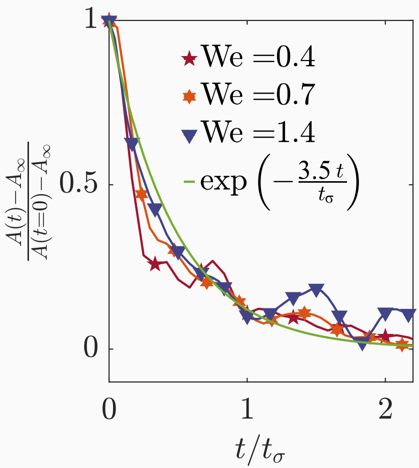
Distribution of energy injection





- Exponentially distributed with $\gamma = 2.6$
- Decorrelates within one eddy turnover time
- Statistical details: Morón et al., arXiv 2026, *Stretching by outer eddies sets the turbulent breakup rate.*

Rate of drop relaxation



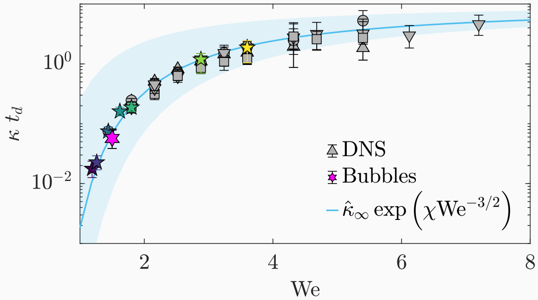
- Initialise the drops as ellipsoids
- Measure relaxation to dynamic equilibrium in the turbulent flow
- Relaxation rate is set by the capillary time scale: $t_\sigma = \sqrt{\frac{\rho d^3}{\sigma}}$

Breakup rate

- Model the interface energy $\mathcal{H} = \sigma A$ as a stochastic variable:

$$\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}t} = \underbrace{\vartheta_O}_{P(\vartheta_O) \propto e^{-2.6\vartheta_O}} - \frac{3.5}{t_\sigma}\left(\mathcal{H} - \mathcal{H}_0\right), \tag{1}$$

- Breakup occurs if $\mathcal{H} > 1.27\mathcal{H}_0$.
- Integrate (**??**) for $1t_d$ to obtain the breakup probability $\hat{\kappa} = 10t_d^{-1} \exp\left(-\chi \mathrm{We}^{-3/2}\right)$
- The breakup rate of bubbles is also accurately predicted [Rivière et al., JFM, 2024].



Conclusions

- Used phase-field DNS to obtain 4D spatio-temporal spectra of drops in turbulence.
- **What size eddies cause deformation?** Drops act as spectral filters, absorbing energy at large scales ($\sim d$) and returning it at small scales.
- **What are the timescales?** Drop relaxation is governed by the capillary time t_σ, while turbulent forcing decorrelates within one eddy turnover time t_d.
- **Mechanistic model?** A stochastic model balancing exponential forcing and capillary relaxation accurately predicts breakup rates for both drops and bubbles.

Thank you. Questions?

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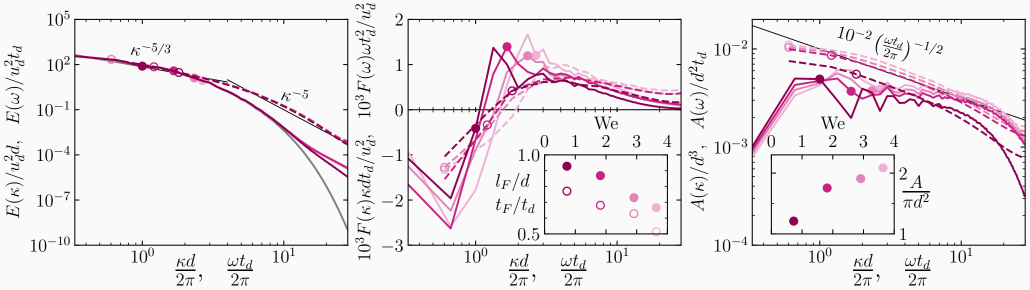
One-dimensional spectra: scaling & crossover



Left: $E(\kappa)$, $E(\omega)$

K41 $\kappa^{-5/3}$ over a decade. Drops $\Rightarrow$ modest small-scale increase, growing with We.

Middle: $F(\kappa)$, $F(\omega)$

Crossover $F = 0$ at length l_F and time t_F. $\mathrm{We} \to 0 \Rightarrow l_F \to d$, $t_F \to t_d$.

Right: $A(\kappa)$, $A(\omega)$

$A(\omega) \sim \omega^{-1/2}$ over much of the range. Lower We $\Rightarrow$ smoother interface.