Turbulence and intermittency in homogeneous isotropic flows of elastoviscoplastic fluids

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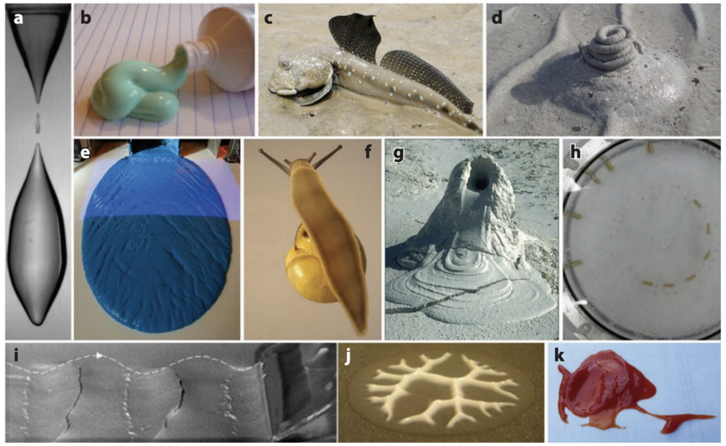
Elastoviscoplastic fluids

Elastoviscoplastic (EVP) fluids
- solid (unyielded) $\tau \leq \tau_y$
- liquid (yielded) $\tau > \tau_y$

With elastic effects present in both solid and liquid states.



Balmforth et al. *Annu. Rev. Fluid Mech.* **46**, 121–146 (2014).

CBC News - Mudslide barrels through Swiss town

https://mycal.com.pe/medio-ambiente/tratamiento-de-lodos/

Previous research in turbulent EVP flows

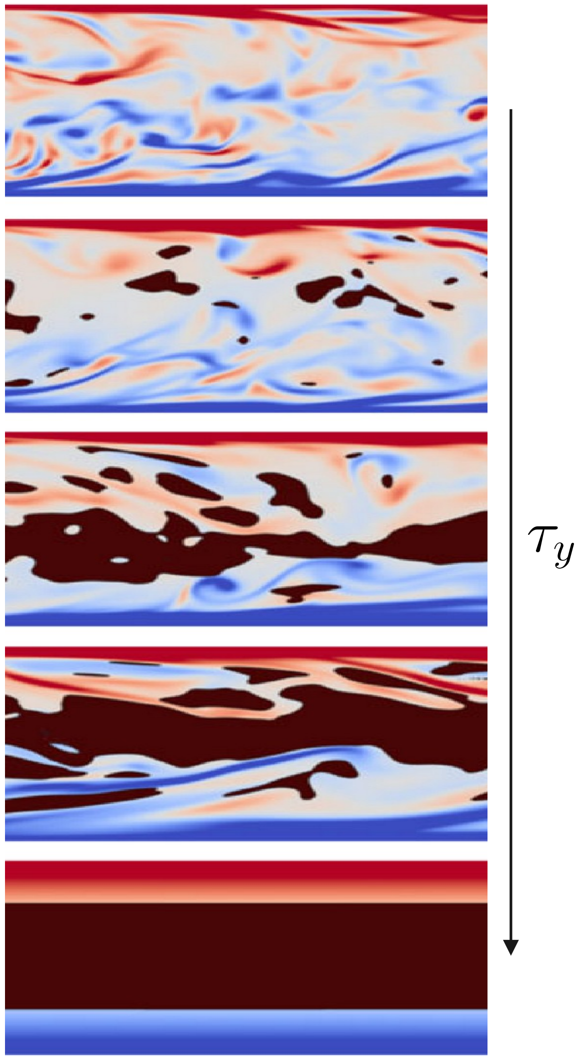
Rosti et al. (JFM 2018) made the **first** numerical simulation of an **EVP turbulent channel flow**

⬆ τ_y ⬆ volume of unyielded region
⬇ drag

Le Clainche et al. (JFM 2020) applied higher order dynamic mode decomposition (HODMD)

⬆ τ_y ⬆ length of streamwise velocity streaks

Mitishita et al. (JNNFM 2021): verified these results experimentally.



Rosti et al. (JFM 2018)

Objective

We study a statistically **homogenous** and **isotropic** flow of an **EVP** fluid

How are Kolmogorov's (1941) predictions affected when the fluid is elastoviscoplastic?

Numerical Setup

Flow equations are solved on a 1024^3 grid with **periodic boundary conditions** in x, y, z.

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = \frac{1}{\rho}\nabla p + \nu\nabla^2\boldsymbol{u} + \nabla\cdot\boldsymbol{\tau} + \boldsymbol{f}_{\text{inj}}$$

$$\nabla\cdot\boldsymbol{u} = 0$$

Saramito (JNNFM 2007) **model**

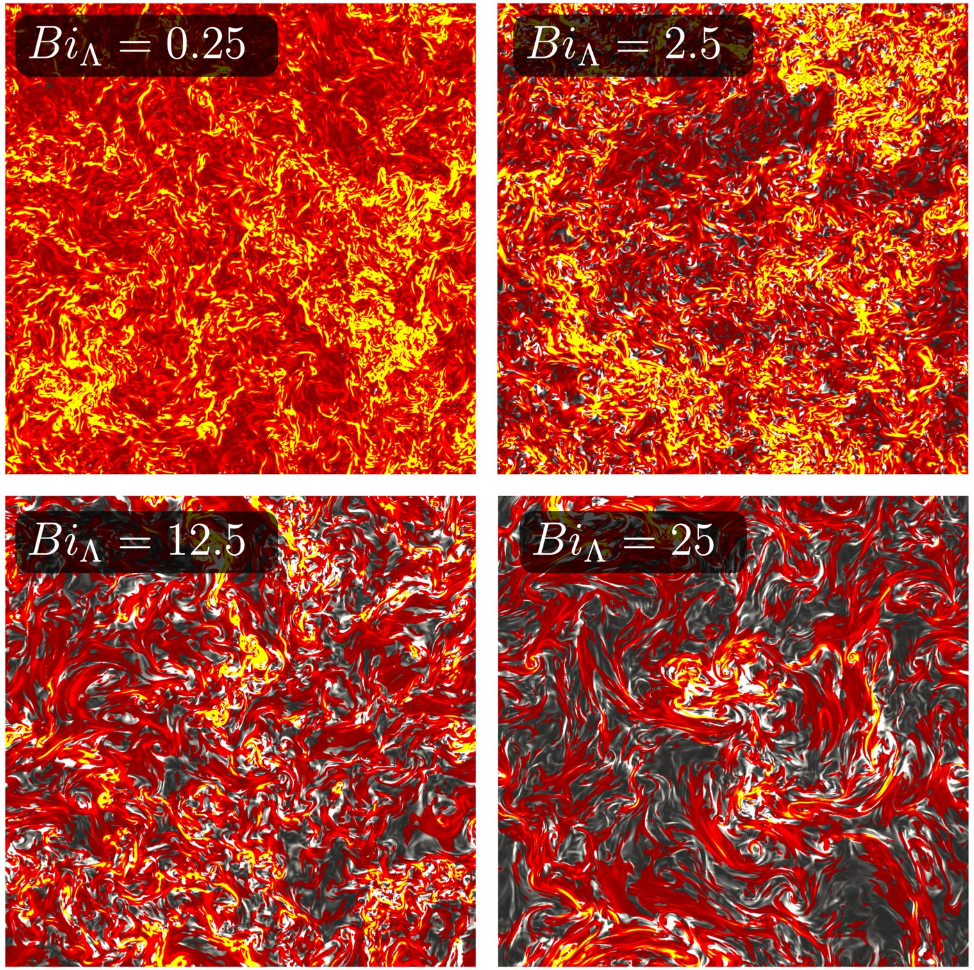
$$\lambda\overset{\nabla}{\boldsymbol{\tau}} + \max\left(0, \frac{\tau_d - \tau_y}{\tau_d}\right)\boldsymbol{\tau} = \eta_p\left(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T\right)$$

$$\overset{\nabla}{\boldsymbol{\tau}} = \frac{\partial\boldsymbol{\tau}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{\tau} - (\nabla\boldsymbol{u})^T\cdot\boldsymbol{\tau} - \boldsymbol{\tau}\cdot\nabla\boldsymbol{u}$$

Dimesionless groups

$$Re_\Lambda \equiv \frac{\rho u'\Lambda}{\eta_0} \approx 435, \quad Wi_\Lambda \equiv \frac{\lambda u'}{\Lambda} << 1,$$

$$0 \leq Bi_\Lambda \equiv \frac{\rho\tau_y\Lambda}{\eta_0 u'} < 25.$$

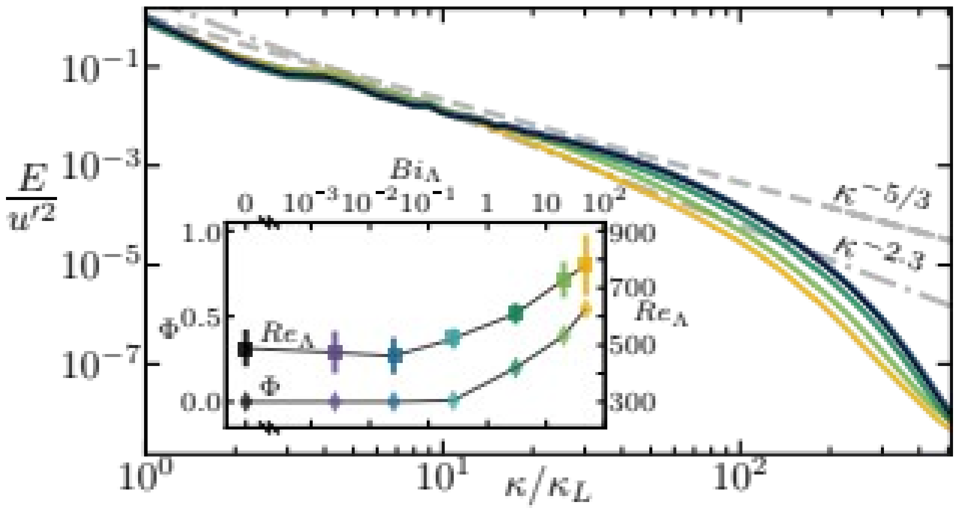


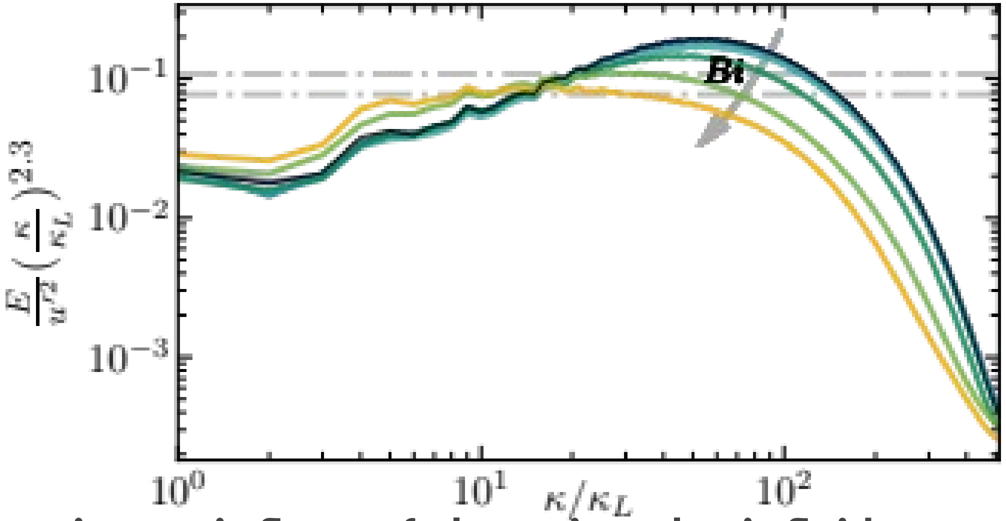
Λ Taylor's micro-scale
λ relaxation time
η_0 viscosity
u' RMS of fluid velocity
τ_d magnitude of deviatoric stress
τ_y yield stress

Effect of plasticity on the kinetic energy spectrum

- Bulk properties (Re_Λ and Φ) increase for $Bi_\Lambda > 1$
- Energy increase at large scales
- Energy decrease at small scales
- Apparent $E \sim \kappa^{-2.3}$ scaling at large Bi_Λ

Bi_Λ= [0, 0.0025, 0.025, 0.25, 2.5, 12.5, 25]





Energy scaling in turbulent flows of viscoelastic fluids:

Rosti et al, *arXiv* **2**, 1–11 (2021).
Zhang et al, *Sci. Adv.* **7**, 1–9 (2021).

Φ volume fraction of the unyielded regions

Scale-by-scale energy balance

The energy flux due to each term in the Navier-Stokes equation is given by the balance:

$$\mathcal{F}_{\mathrm{inj}}(\kappa) + \Pi(\kappa) + \mathcal{D}(\kappa) + \mathcal{N}(\kappa) = \langle \epsilon_{\mathrm{f}} \rangle + \langle \epsilon_{\mathrm{n}} \rangle = \langle \epsilon_{\mathrm{t}} \rangle,$$

Turbulent forcing

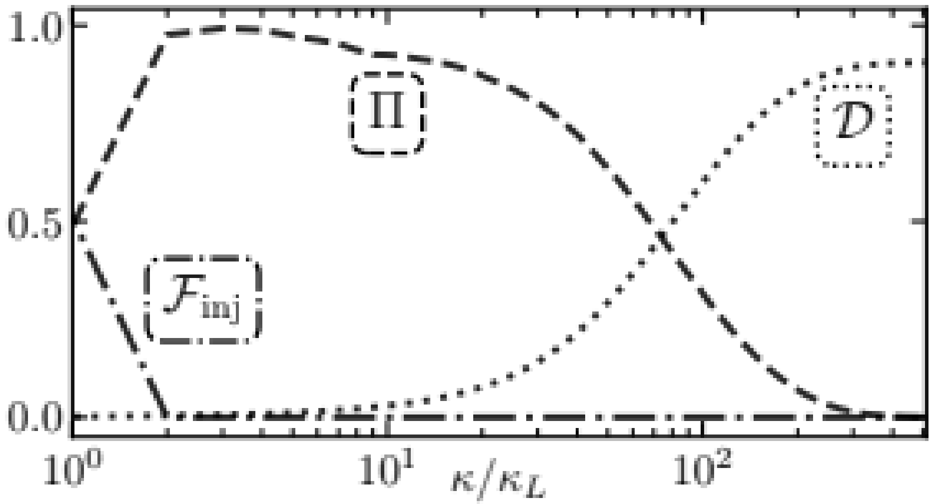
Convection

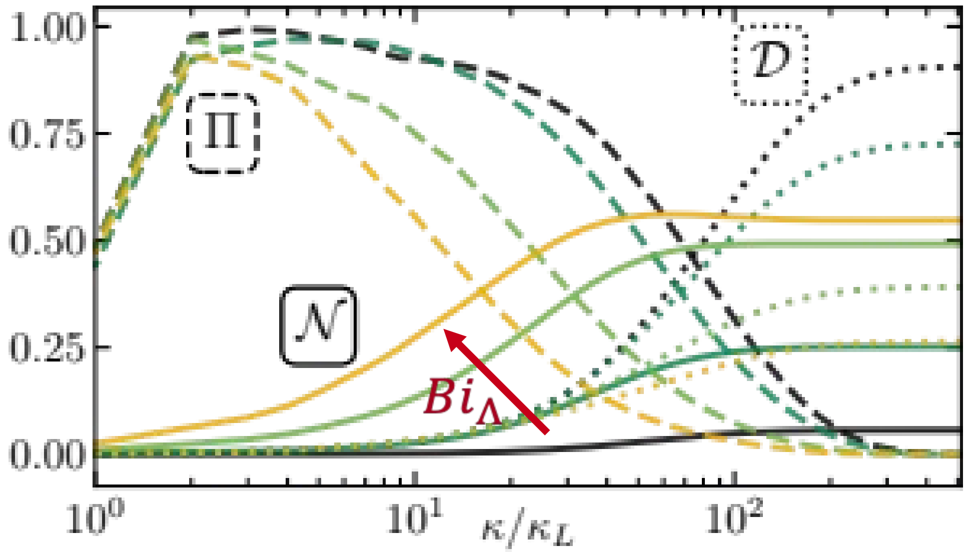
Fluid dissipation

Non-Newtonian

Bi_Λ= [0, 0.0025, 0.025, 0.25, 2.5, 12.5, 25]

- Non-Newtonian contribution ⬆️ as plasticity ⬆️





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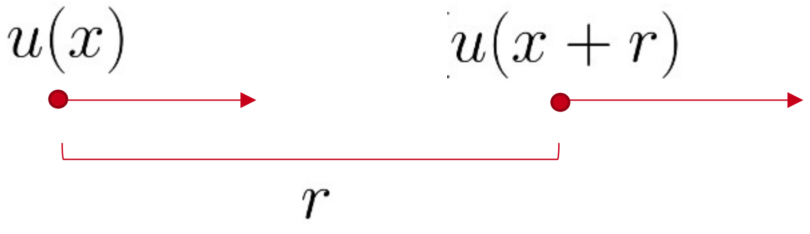
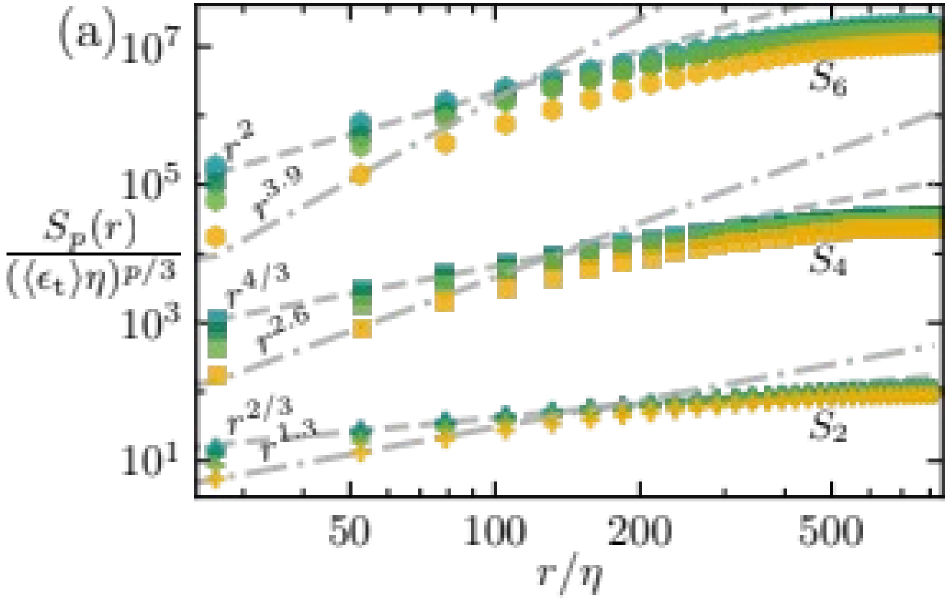
Structure functions and intermittency

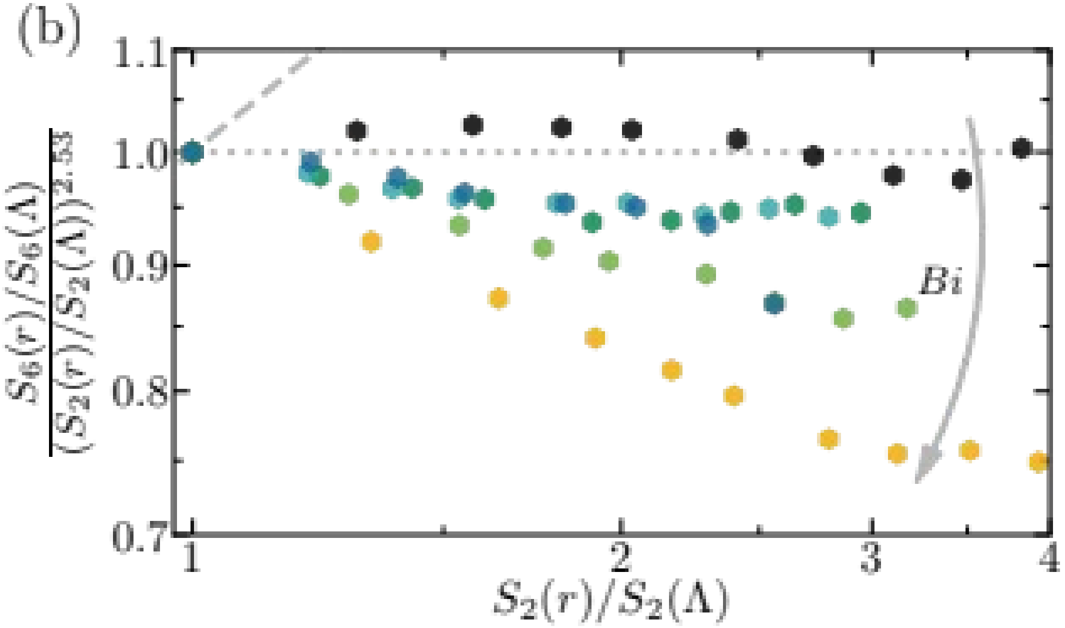
- Structure functions allow us to test Kolmogorov's hypotheses

$$S_{\mathrm{p}}(r) = \langle [u(x+r) - u(x)]^p \rangle \ \sim (\langle \epsilon_{\mathrm{t}} \rangle r)^{p/3}$$



$\Longrightarrow$ Intermittency grows as Bi_Λ increases





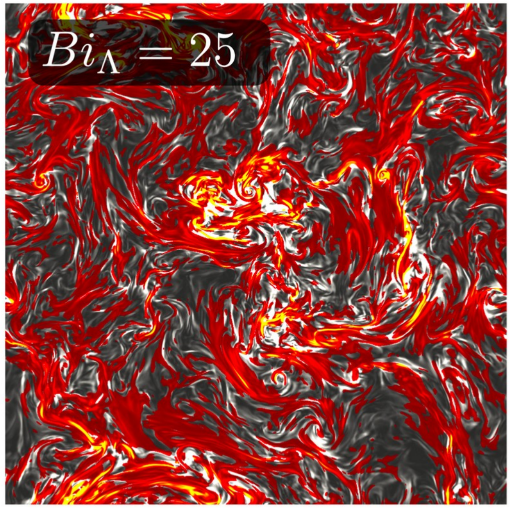
Bi_Λ= [0, 0.0025, 0.025, 0.25, 2.5, 12.5, 25]

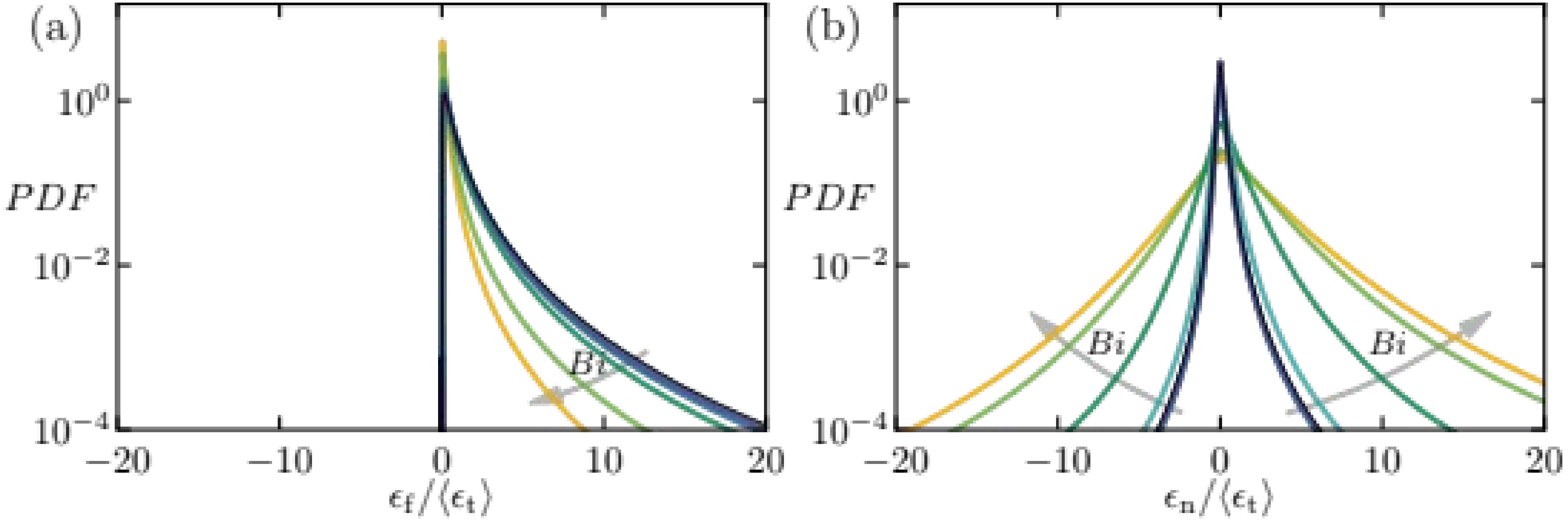
Kolmogorov, *J. Fluid Mech.* **13**, 82–85 (1962).
Benzi et al, *Phys. Rev. E.* **48**, 29-32 (1993).
Mandelbrot, *J. Fluid Mech.* **62**, 331–358 (1974).
Frisch, *Cambridge Uni. Press* (1995).

Intermittency of the dissipation

- Turbulent kinetic energy is dissipated by the fluid and the non-Newtonian stresses; $\epsilon_t = \epsilon_f + \epsilon_n$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = \frac{1}{\rho}\nabla p + \overbrace{\nu \nabla^2 \boldsymbol{u}}^{\epsilon_f} + \overbrace{\nabla \cdot \boldsymbol{\tau}}^{\epsilon_n} + \boldsymbol{f}_{\text{inj}}$$





The distribution of ϵ_f narrows and of ϵ_n significantly broadens as Bi_Λ increases.

$\Longrightarrow$ The extreme values of ϵ_n are the source of the enhanced intermittency.

Bi_Λ= [0, 0.0025, 0.025, 0.25, 2.5, 12.5, 25]

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Conclusion

By means of unprecedented high-Reynolds-number DNS of an elastoviscoplastic fluid, we have shown that:

- Plastic effects significantly alter the classical turbulence predicted by the Kolmogorov theory for Newtonian fluids
- The non-Newtonian contribution to the energy balance becomes dominant at intermediate and small scales for large Bingham numbers, inducing the emergence of a new intermediate scaling range in the energy spectra between the Kolmogorov inertial and the dissipative ranges, where energy spectrum decays with a -2.3 exponent
- The flow in the presence of plastic effects is more intermittent than in a Newtonian fluid, due to the dissipation rate, which includes a new plastic contribution that grows with the Bingham number

Thank you!
ありがとうございます！

Numerical Setup

Homogeneous isotropic turbulence (HIT)
with $Re_\Lambda \approx 435$ on a 1024^3 grid

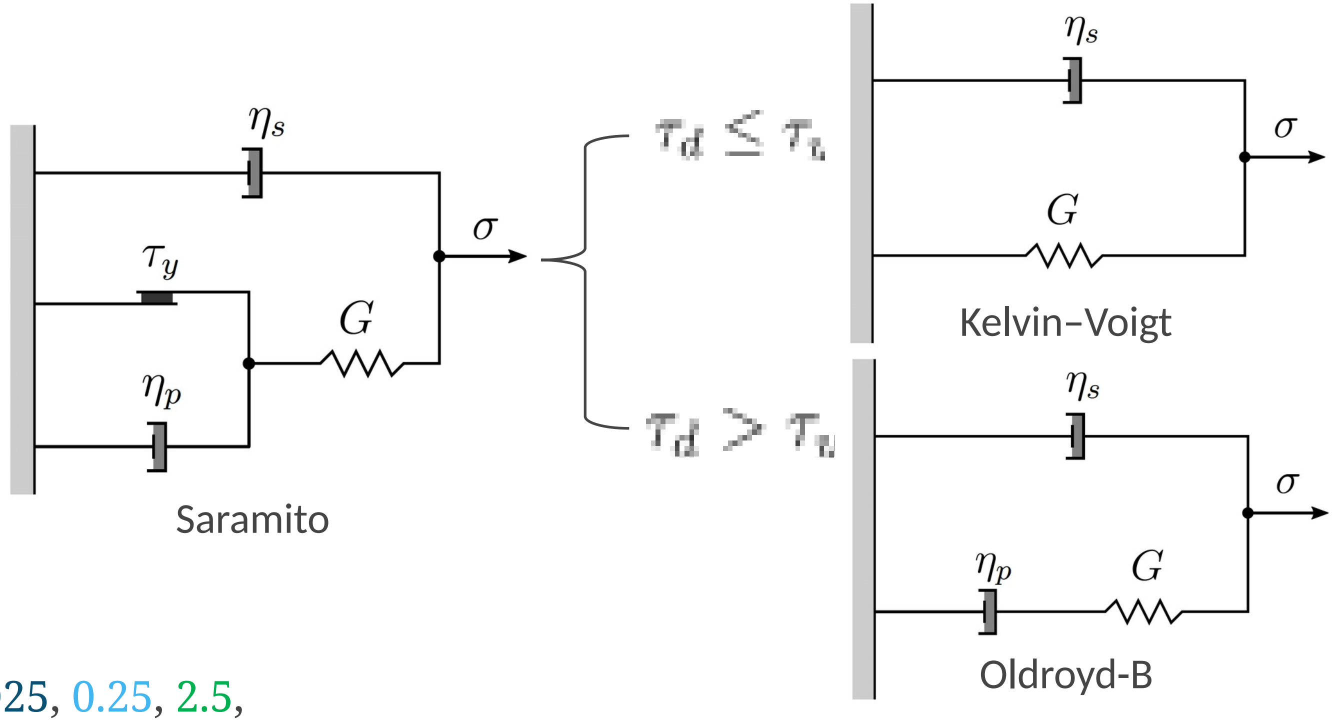
$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = \frac{1}{\rho}\nabla p + \nu\nabla^2\boldsymbol{u} + \boldsymbol{f}_{\text{evp}} + \boldsymbol{f}_{\text{inj}} \qquad \boldsymbol{f}_{\text{evp}} \equiv \nabla\cdot\boldsymbol{\tau}/\rho$$

$$\nabla\cdot\boldsymbol{u} = 0$$

Saramito model (2007)

$$\lambda\overset{\nabla}{\boldsymbol{\tau}} + \max\left(0, \frac{\tau_d - \tau_y}{\tau_d}\right)\boldsymbol{\tau} = \eta_p\left(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T\right)$$

$$\boldsymbol{\tau}_d = \boldsymbol{\tau} - \frac{1}{N}\mathrm{tr}(\boldsymbol{\tau})\boldsymbol{I}$$



Dimesionless groups

$$\alpha \equiv \frac{\eta_p}{\eta_0} = 0.1, \eta_0 = \eta_s + \eta_p$$

$$Re_\Lambda \equiv \frac{\rho u'\Lambda}{\eta_0}, \; Wi_\Lambda \equiv \frac{\lambda u'}{\Lambda} << 1$$

We study the effect of $Bi_\Lambda \equiv \frac{\rho\tau_y\Lambda}{\eta_0 u'} = [0, 0.0025, 0.025, 0.25, 2.5, 12.5, 25]$

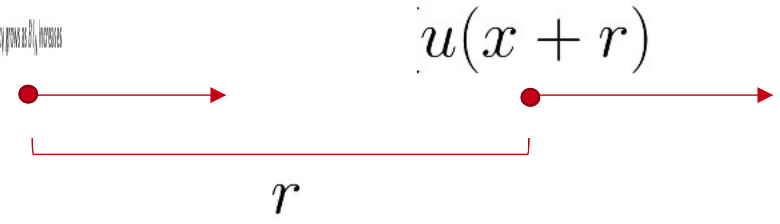
Λ Taylor's micro-scale
λ relaxation time
η_s fluid viscosity
η_p Non-Newtonian viscosity
τ_d magnitude of deviatoric stress tensor
G elastic modulus

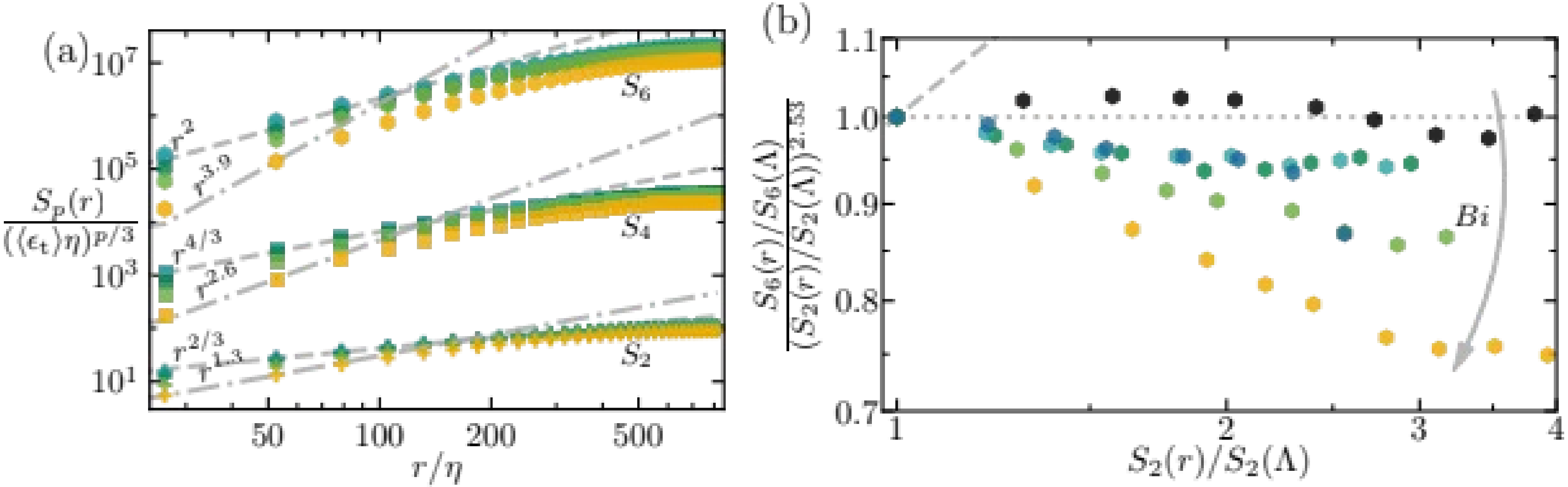
Structure functions and intermittency

- Structure functions allow us to test Kolmogorov's hypotheses

$$S_\mathrm{p}(r) = \langle [u(x+r) - u(x)]^p \rangle \ \sim ((\langle \epsilon_\mathrm{t} \rangle r)^{p/3} A_p$$

$\implies$ Intermittency grows as Bi_Λ increases





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Frisch, *Cambridge Uni. Press* (1995).

Effect of elasticity

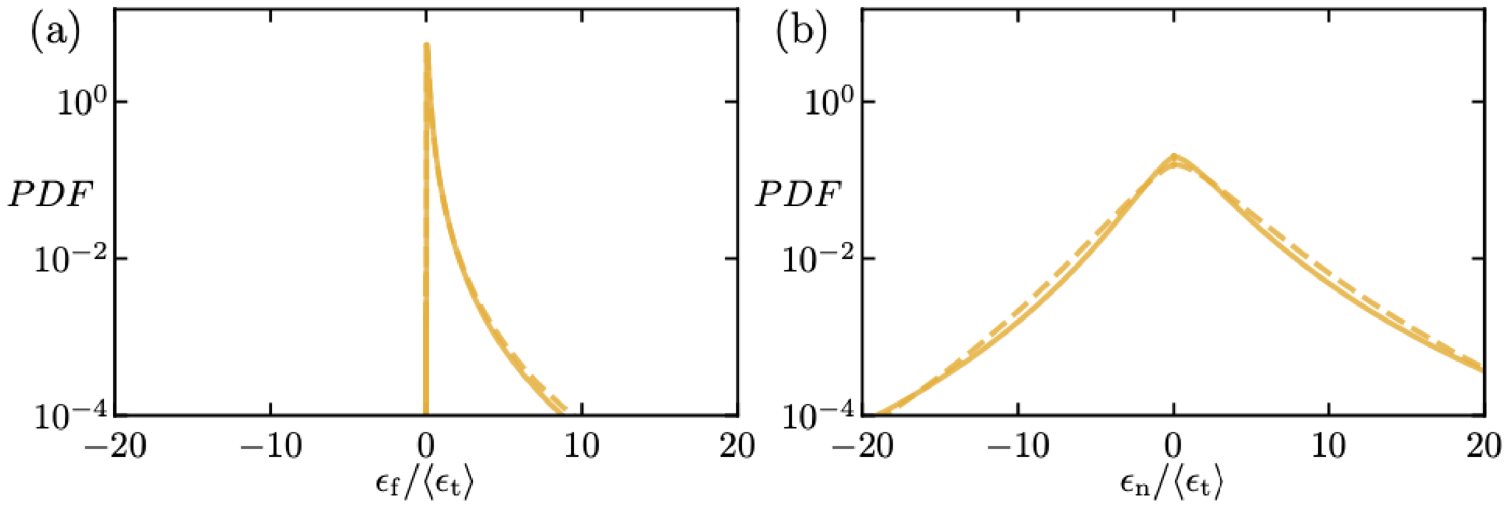


FIG. S2. Probability distribution function (PDF) of (a) the fluid dissipation rate ϵ_f and of (b) the non-Newtonian dissipation rate ϵ_n averaged over time for $Bi_\Lambda = 25$ at $Wi_\Lambda = 0.01$ (solid line) and $Wi_\Lambda = 0.001$ (dashed line).

Energy fluxes

$$\frac{\mathrm{d}\hat{\boldsymbol{u}}}{\mathrm{d}t} + \hat{\boldsymbol{G}} = -i\boldsymbol{\kappa}\frac{\hat{p}}{\rho} - \nu\kappa^2\hat{\boldsymbol{u}} + \hat{\boldsymbol{f}}_{\mathrm{inj}} + \hat{\boldsymbol{f}}_{\mathrm{evp}}, \tag{S6}$$

where $\hat{G}$ is the Fourier coefficient of the non-linear convective term appearing in Eq. (S2), and i is the imaginary unit. Similar equations can be obtained for the complex conjugate $\hat{\boldsymbol{u}}^*$. When Eq. (S6) is multiplied by $\hat{\boldsymbol{u}}^*$, the pressure term $-i\boldsymbol{\kappa}\cdot\hat{\boldsymbol{u}}^*\frac{\hat{p}}{\rho}$ vanishes due to the incompressibility constraint (Eq. (S5)), and the viscous term $-\nu\kappa^2\hat{\boldsymbol{u}}\cdot\hat{\boldsymbol{u}}^*$ can be expressed in terms of the kinetic energy; $-2\nu\kappa^2\hat{E}$. The same holds when multiplying the momentum equation of $\hat{\boldsymbol{u}}^*$ by $\hat{\boldsymbol{u}}$. By summing the two equations for $\hat{\boldsymbol{u}}$ and $\hat{\boldsymbol{u}}^*$ and dividing by 2, we have an expression for the time evolution of turbulent kinetic energy $\hat{E}(\boldsymbol{\kappa}, t)$

$$\frac{\mathrm{d}\hat{E}(\boldsymbol{\kappa})}{\mathrm{d}t} = \hat{T}(\boldsymbol{\kappa}) + \hat{V}(\boldsymbol{\kappa}) + \hat{F}_{\mathrm{inj}}(\boldsymbol{\kappa}) + \hat{F}_{\mathrm{evp}}(\boldsymbol{\kappa}), \tag{S7}$$

where the terms on the right-hand side represent the following contributions: $\hat{T} = -\frac{1}{2}(\hat{\boldsymbol{G}}\cdot\hat{\boldsymbol{u}}^* + \hat{\boldsymbol{G}}^*\cdot\hat{\boldsymbol{u}})$ is due to the non-linear convective term, $\hat{V} = -2\nu\kappa^2\hat{E}$ is due to the fluid dissipation term, $\hat{F}_{\mathrm{inj}} = \frac{1}{2}(\hat{\boldsymbol{f}}_{\mathrm{inj}}\cdot\hat{\boldsymbol{u}}^* + \hat{\boldsymbol{f}}^*_{\mathrm{inj}}\cdot\hat{\boldsymbol{u}})$ is due to the external forcing, and $\hat{F}_{\mathrm{evp}} = \frac{1}{2}(\hat{\boldsymbol{f}}_{\mathrm{evp}}\cdot\hat{\boldsymbol{u}}^* + \hat{\boldsymbol{f}}^*_{\mathrm{evp}}\cdot\hat{\boldsymbol{u}})$ is due to the non-Newtonian stress. The one-dimensional energy spectrum $E(\kappa, t)$ can be obtained by isotropically averaging Eq. (S7) over the sphere of radius κ (i.e., $E(\kappa,t) = \iint_{S(\kappa)}\hat{E}(\boldsymbol{\kappa},t)\mathrm{d}S(\kappa)$, where $S(\kappa)$ is the sphere defined by $\boldsymbol{\kappa}\cdot\boldsymbol{\kappa} = \kappa^2$),

$$\frac{\mathrm{d}E(\kappa)}{\mathrm{d}t} = T(\kappa) + V(\kappa) + F_{\mathrm{inj}}(\kappa) + F_{\mathrm{evp}}(\kappa). \tag{S8}$$

where $\frac{\mathrm{d}E}{\mathrm{d}t}$ becomes zero for a statistically stationary flow. Integrating Eq. (S8) from κ to infinity, we obtain the energy-transfer balance

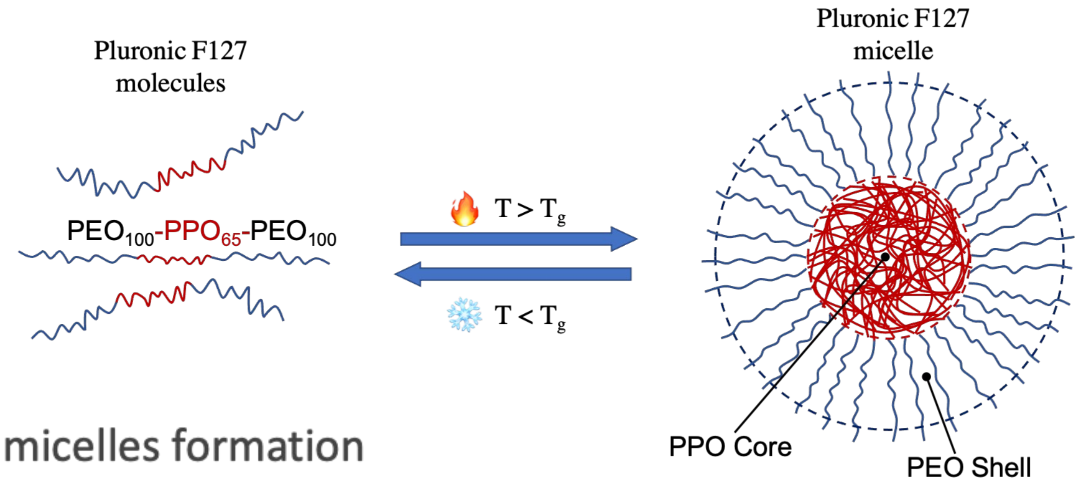
$$0 = \Pi + \mathcal{D}' + \mathcal{F}_{\mathrm{inj}} + \mathcal{N}', \tag{S9}$$

where $\Pi(\kappa) \equiv \int_\kappa^\infty T(\kappa)\,\mathrm{d}\kappa$, $\mathcal{D}'(\kappa) \equiv \int_\kappa^\infty V(\kappa)\,\mathrm{d}\kappa$, $\mathcal{F}_{\mathrm{inj}}(\kappa) \equiv \int_\kappa^\infty F_{\mathrm{inj}}(\kappa)\,\mathrm{d}\kappa$, and $\mathcal{N}'(\kappa) \equiv \int_\kappa^\infty F_{\mathrm{evp}}(\kappa)\,\mathrm{d}\kappa$ represent the contributions to the spectral power balance from the non-linear convective, fluid dissipation, turbulence forcing, and non-Newtonian terms, respectively. The fluid dissipation term can be expressed as $\mathcal{D}(\kappa) = -\int_0^\kappa V(\kappa)\,\mathrm{d}\kappa = \mathcal{D}'(\kappa) + \epsilon_{\mathrm{f}}$, where $\epsilon_{\mathrm{f}} = -\int_0^\infty V(\kappa)\,\mathrm{d}\kappa$ is the energy dissipated by the fluid viscosity. Similarly, the non-Newtonian contribution can be written as $\mathcal{N}(\kappa) = -\int_0^\kappa F_{\mathrm{evp}}(\kappa)\,\mathrm{d}\kappa = \mathcal{N}'(\kappa) + \epsilon_{\mathrm{p}}$, where $\epsilon_{\mathrm{p}} = -\int_0^\infty F_{\mathrm{evp}}(\kappa)\,\mathrm{d}\kappa$ is the non-Newtonian dissipation. Substituting this in Eq. (S9), we obtain the energy balance equation (Eq. (1)) used in the main letter.

Elastoviscoplastic material

Pf127 solution of 20 wt % *

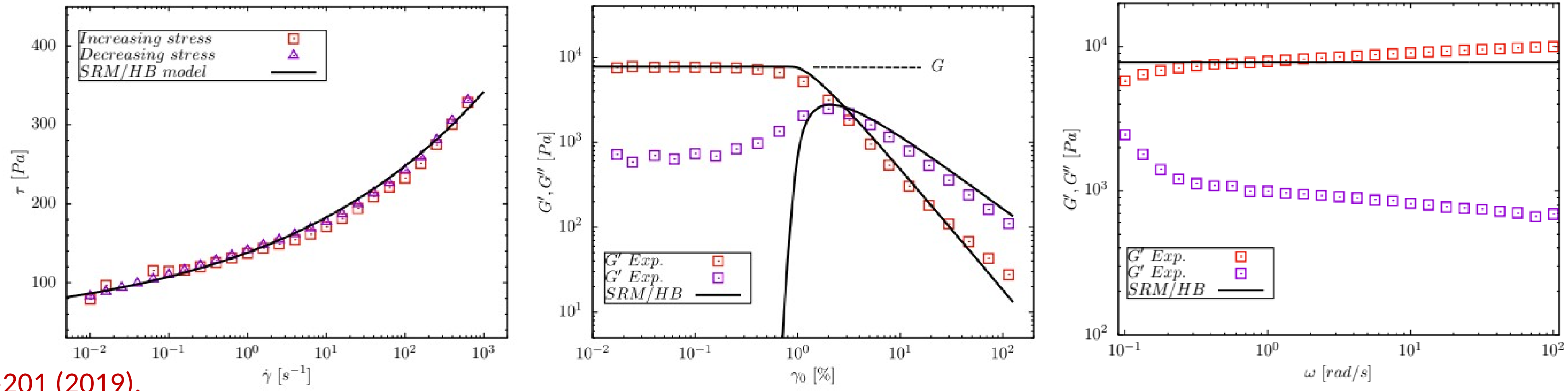
- At low temperatures (T) and concerations (C):
  PEO and PPO are soluble in water
- Above $C_{crit} \approx 12$ wt % as T increases:
  Hyprophobicity/solubality of PPO blocks decreases → micelles formation



Rheological measurements**

Model parameters:

$$\tau_y = 40 \text{ Pa}$$
$$n = 0.163$$
$$k = 98 \text{ Pa.s}^n$$
$$G = 7800 \text{ Pa}$$



*Hopkins et al. *J. Rheol.* **63**, 191–201 (2019).
** Varchanis *et al. PNAS.* 201922242 (2020).