Droplet morphology in turbulence

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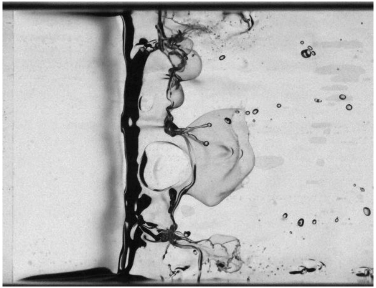
Morphology of clean and surfactant-laden droplets in turbulent flows

CO_2 absorption[a]

Wave breaking on the coast of Okinawa[b]

Droplet breakup during cough[c]



1mm water & glycerol film with 15m/s airflow.

[a] Merlivat, L., & Memery, L. (1983). *J. Geophys. Res.-Oceans*, (C1)
[b] Photo credit: Linda Gietka
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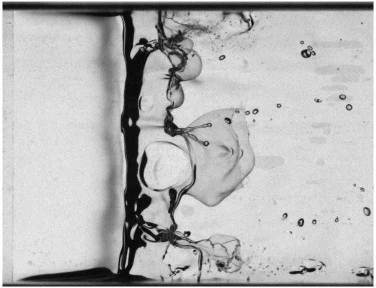

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What is the morphology of droplets in turbulence?

How do surfactants affect morphology?

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Numerical method

We use the in-house solver *Fujin*.

Fluid motion[d],

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla \cdot \left[\mu \left(\nabla \mathbf{u} + \nabla \mathbf{u}^T \right) \right] + \nabla \cdot (\boldsymbol{\tau}_c f_\sigma) + \mathbf{f}_s,$$
$$\nabla \cdot \mathbf{u} = 0, \qquad (1)$$

Droplet motion[e],

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\mathbf{u}\phi) = 0, \qquad (2)$$

Surfactant motion[f],

$$\frac{\partial \psi}{\partial t} + \mathbf{u} \cdot \nabla \psi = \nabla \cdot (M_\psi \nabla \mu_\psi), \qquad (3)$$

Surface tension[g],

$$f_\sigma = \max[\sigma_{min}, \sigma_0(1 + \beta_s \log(1 - \psi))]. \qquad (4)$$

[d] Eswaran, V., & Pope, S. B. (1988). *Computers & Fluids*, (3).

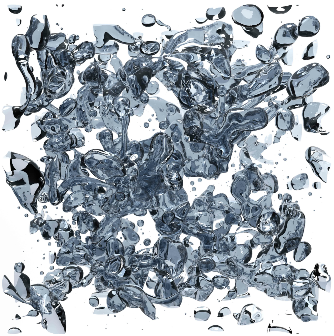
[e] Ii, S., Sugiyama, K., Takeuchi, S., Takagi, S., Matsumoto, Y., & Xiao, F. (2012). *Journal of Computational Physics*, (5).

[f] Engblom, S., Do-Quang, M., Amberg, G., & Tornberg, A. (2013). *Commun. Comput. Phys.*, (4).

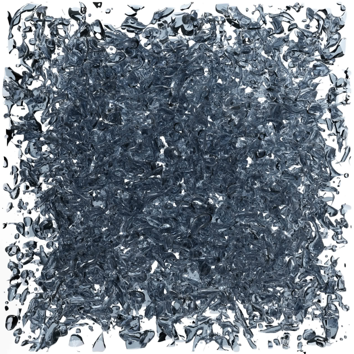
[g] Muradoglu, M., & Tryggvason, G. (2008). *Journal of Computational Physics*, (4).

Setup

- Tri-periodic domain with $500 \times 500 \times 500$ grid points.
- Six simulations with three values of surface tension, and two surfactant concentrations.
- Hinze-scale[h] $d_H \equiv 0.725\sigma^{3/5}\rho^{-3/5}\epsilon^{-2/5}$.



d_H



d_H

case		$\langle\phi\rangle$	$\langle\psi\rangle$	We
SP	●	0	0	-
C10	●	0.1	0	10
C20	●	0.1	0	20
C40	●	0.1	0	40
S05	◆	0.1	0.1	5
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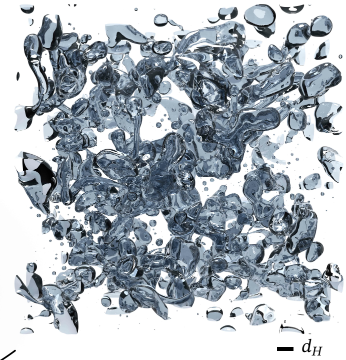

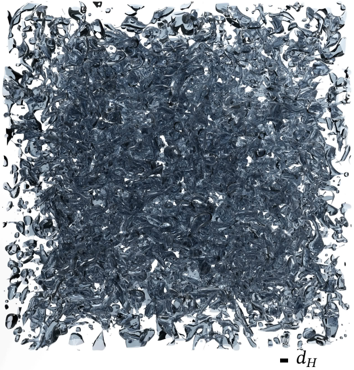

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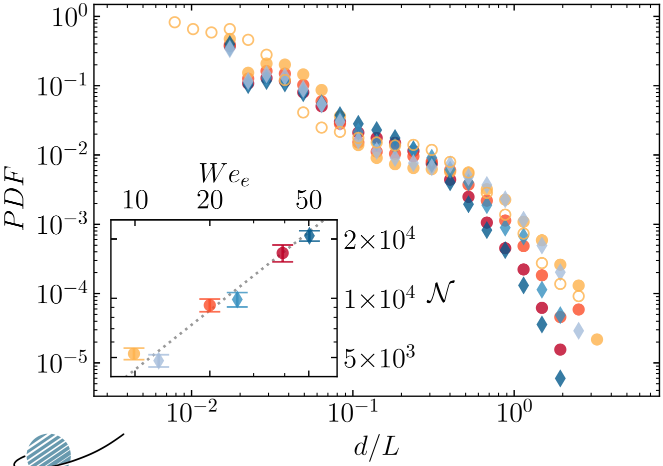
Counting droplets



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- We use a flood-fill algorithm to count the droplets[a].
- Surface tension $\sigma\downarrow$, number of droplets $\mathcal{N}\uparrow$.
- Define a volume-based diameter for each droplet, $d \equiv (6V/\pi)^{1/3}$.

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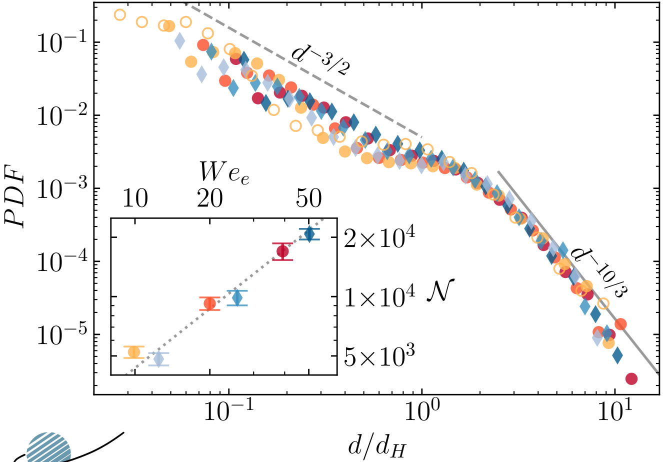

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- Surface tension $\sigma \downarrow$, number of droplets $\mathcal{N} \uparrow$.
- Define a volume-based diameter for each droplet, $d \equiv (6V/\pi)^{1/3}$.
- $d < d_H$: coalescence.
- $d > d_H$: breakup[b].

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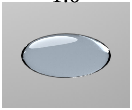
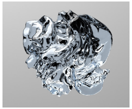
What shape are the droplets?





Aspect ratio $\sqrt{I_1/I_3}$	0.946	0.633	0.905
Sphericity d/d_A	0.999	0.965	0.701

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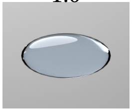
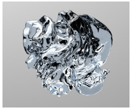
What shape are the droplets?

←1.0→ ↑ 0.9 ↓

←1.0→ ↕ 0.5





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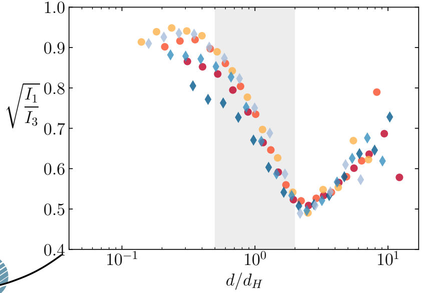

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- Area-based diameter for each droplet, $d \equiv \sqrt{A/\pi}$.
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- $d >> d_H$ aspect ratio → 1.

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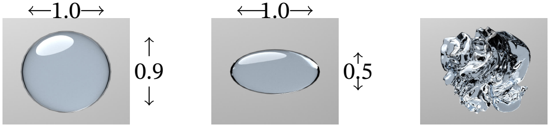

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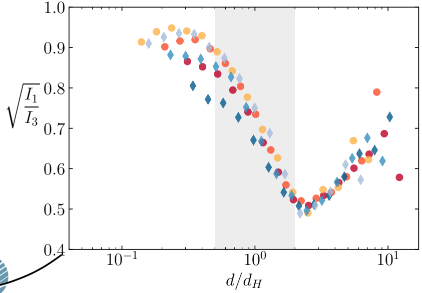

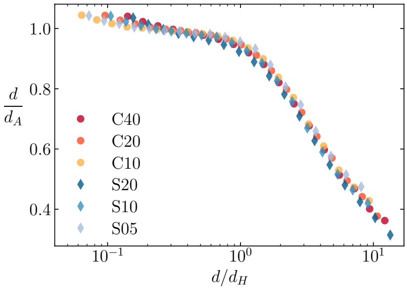

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- $d << d_H$ aspect ratio → 1.
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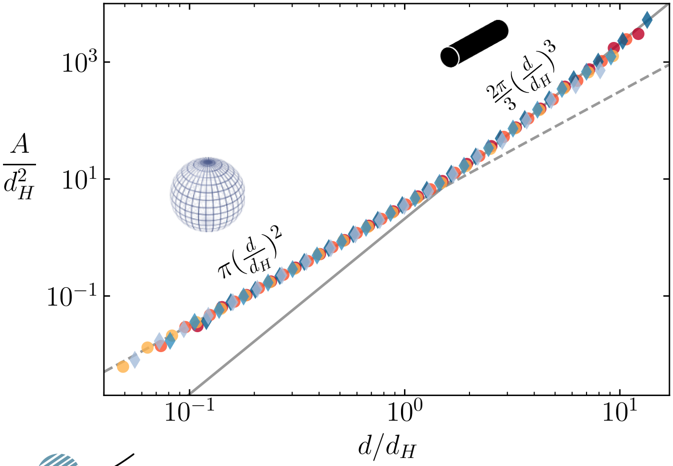
- $d >> d_H$ aspect ratio → 1.
- $d > d_H$ area ↑.

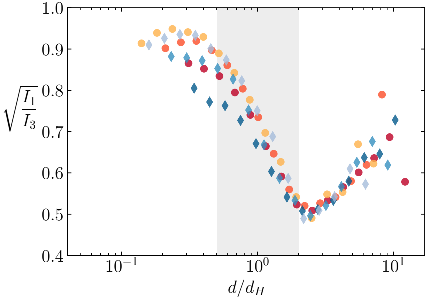
What is the area of the droplets?



- $d < d_H$: spheroids.
- $d > d_H$: filaments with diameter d_H.

Recap

- The large droplets are filamentous.



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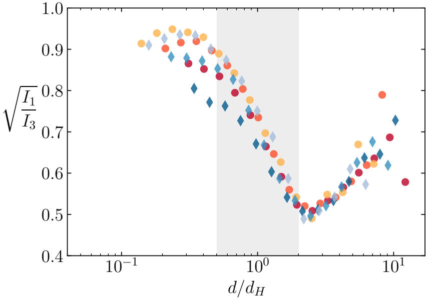

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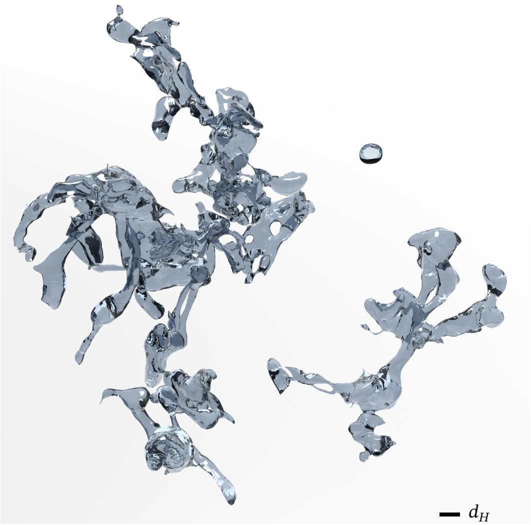
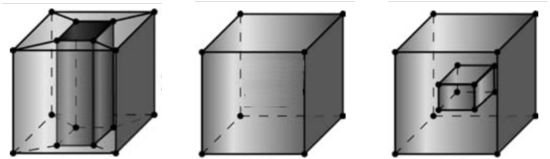


How to count the number of handles?

We compute the Euler characteristic[i] of the droplet interfaces.

$$\chi = n - e + f \tag{5}$$



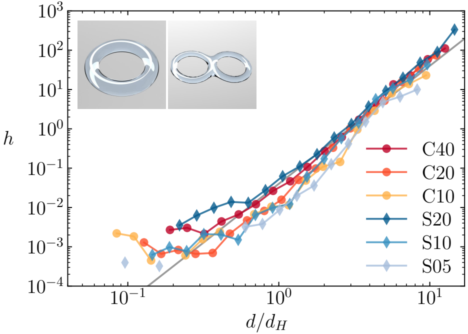
$\chi = 0$ $\chi = 2$ $\chi = 4$

Thus obtaining[j] the number of handles h and voids v.

$$\chi = 2 + 2v - 2h. \tag{6}$$

[i] Massey, W. S. (1997). Springer.
[j] Mendoza, R., Thornton, K., Savin, I., & Voorhees, P. (2006). *Acta Mater.*, (3).

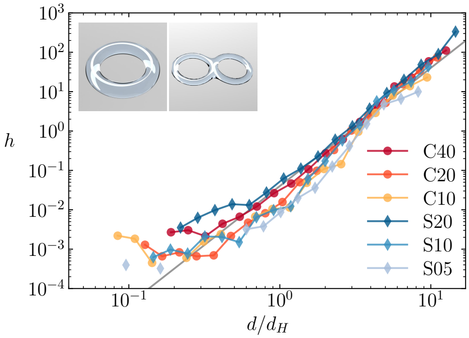
How many handles and voids are there?



$$h = 0.04(d/d_H)^3$$

- Constant number of handles per unit length of filament.

How many handles and voids are there?



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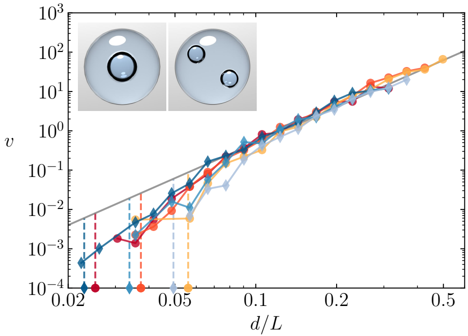
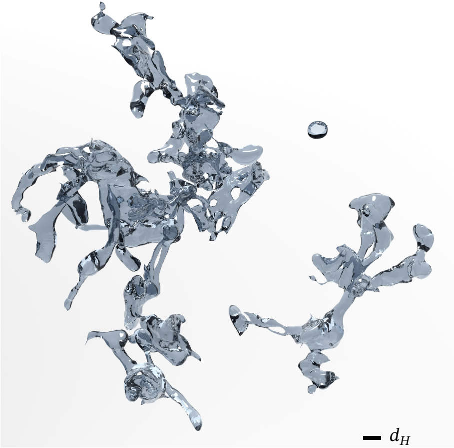


$$v = 500(d/L)^3$$

- Constant number of voids per unit volume of droplet.

Conclusions

- Average surface tension can be used to define the Hinze scale d_H in surfactant-laden flows.



[j] Cannon, I., Soligo, G., & Rosti, M. E. (2024). *Journal of Fluid Mechanics*

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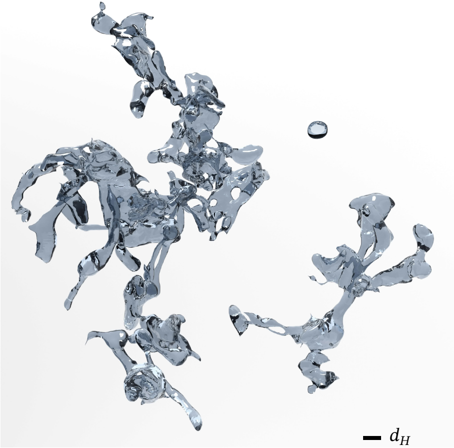

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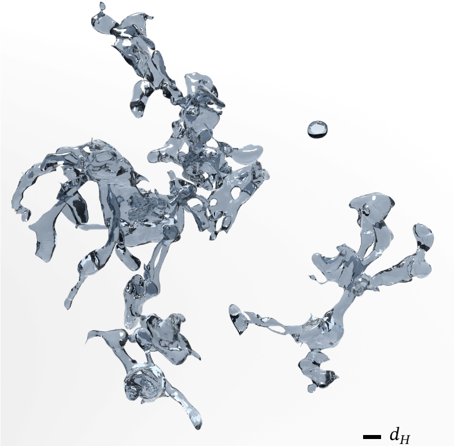

[j]Cannon, I., Soligo, G., & Rosti, M. E. (2024). *Journal of Fluid Mechanics*

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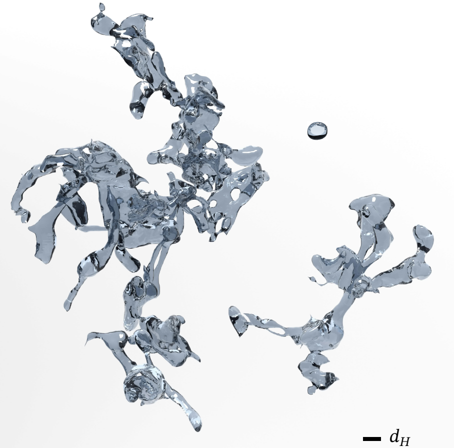
Thank you!



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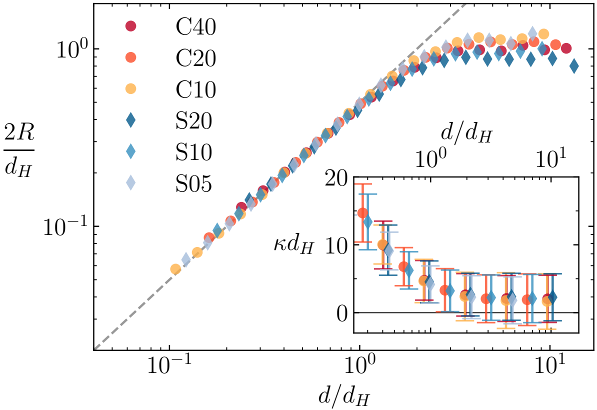
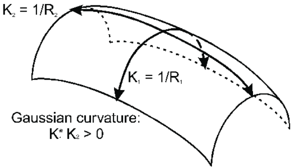
Interface curvature

Curvature $\kappa = \kappa_1 + \kappa_2 = 1/R$.





- $d < d_H$: $R = d/4$.
- $d > d_H$: $R = d_H/2$, with regions of negative curvature.