# Morphology and topology of droplets in turbulence

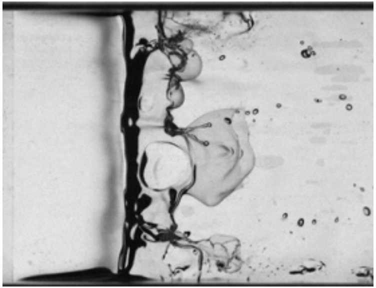
Current advances in turbulence and multiphase flows, 2024 April 16

**Ianto Cannon, Giovanni Soligo & Marco E. Rosti**
Complex Fluids and Flows unit
Okinawa Institute of Science and Technology

$CO_2$ absorption[a]

Wave breaking on the coast of Okinawa[b]

Droplet breakup during cough[c]



1mm water & glycerol film with 15m/s airflow.

What is the morphology of droplets in turbulence?

How do surfactants affect morphology?

[a] Merlivat, L., & Memery, L. (1983). *J. Geophys. Res.-Oceans*, (C1)

[b] Photo credit: Linda Gietka

[c] Kant, P., Pairetti, C., Saade, Y., Popinet, S., Zaleski, S., & Lohse, D. (2023). *Physical Review Fluids*, (7)

# Numerical method

We use the in-house solver *Fujin*.
Fluid motion,

$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho\mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nabla\cdot\left[\mu\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right)\right] + \nabla\cdot(\boldsymbol{\tau}_{\mathbf{c}} f_\sigma) + \mathbf{f}_{\mathbf{s}},$$
$$\nabla\cdot\mathbf{u} = 0, \tag{1}$$

Droplet motion[d],

$$\frac{\partial \phi}{\partial t} + \nabla\cdot(\mathbf{u}\phi) = 0, \tag{2}$$

Surfactant motion[e],

$$\frac{\partial \psi}{\partial t} + \mathbf{u}\cdot\nabla\psi = \nabla\cdot(M_\psi\nabla\mu_\psi), \tag{3}$$

Surface tension[f],

$$f_\sigma = \max[\sigma_{min}, \sigma_0(1 + \beta_s \log(1-\psi))]. \tag{4}$$

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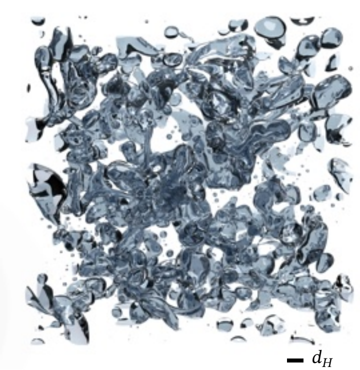
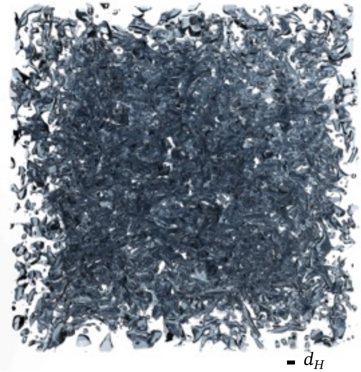
[d] Ii, S., Sugiyama, K., Takeuchi, S., Takagi, S., Matsumoto, Y., & Xiao, F. (2012). *Journal of Computational Physics, (5)*.
[e] Engblom, S., Do-Quang, M., Amberg, G., & Tornberg, A. (2013). *Commun. Comput. Phys., (4)*.
[f] Muradoglu, M., & Tryggvason, G. (2008). *Journal of Computational Physics, (4)*.

# Setup

- Tri-periodic domain with $500 \times 500 \times 500$ grid points.
- Six simulations with three values of surface tension, and two surfactant concentrations.
- Hinze-scale[g] $d_H = 0.725\langle\sigma\rangle_I^{3/5}\rho^{-3/5}\epsilon^{-2/5}$.



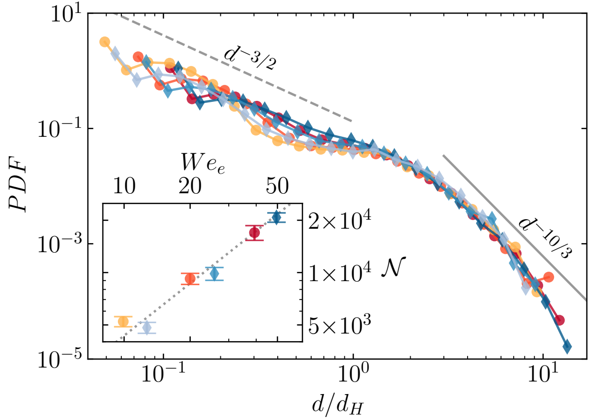




| case | | $\langle\phi\rangle$ | $\langle\psi\rangle$ | $We$ |
|---|---|---|---|---|
| SP | ● | 0 | 0 | - |
| C10 | ● | 0.1 | 0 | 10 |
| C20 | ● | 0.1 | 0 | 20 |
| C40 | ● | 0.1 | 0 | 40 |
| S05 | ◆ | 0.1 | 0.1 | 5 |
| S10 | ◆ | 0.1 | 0.1 | 10 |
| S20 | ◆ | 0.1 | 0.1 | 20 |

- Weber number $We = \rho u'^2 L_b/\sigma$.
- Effective Weber number $We_e \equiv \rho u'^2 L_b/\langle\sigma\rangle_I$.

[g] Hinze, J. O. (1955). *AIChE Journal*, (3).

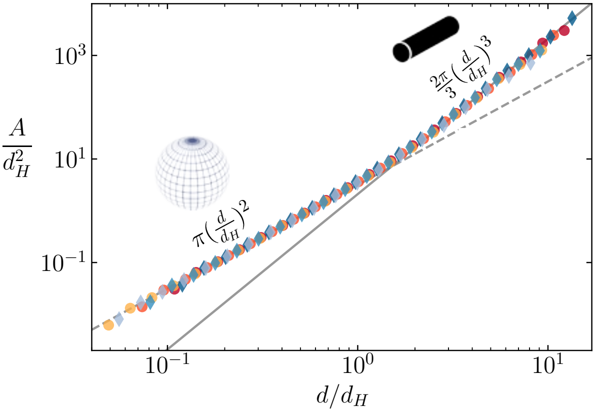
# Counting droplets



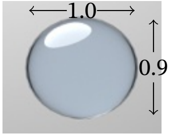

| case | | $\langle\phi\rangle$ | $\langle\psi\rangle$ | $We$ |
|---|---|---|---|---|
| SP | ● | 0 | 0 | - |
| C10 | ● | 0.1 | 0 | 10 |
| C20 | ● | 0.1 | 0 | 20 |
| C40 | ● | 0.1 | 0 | 40 |
| S05 | ◆ | 0.1 | 0.1 | 5 |
| S10 | ◆ | 0.1 | 0.1 | 10 |
| S20 | ◆ | 0.1 | 0.1 | 20 |

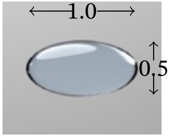
- We use a flood-fill algorithm to count the droplets[a].
- Surface tension $\sigma \downarrow$, number of droplets $\mathcal{N} \uparrow$.
- Define a volume-based diameter for each droplet, $d \equiv (6V/\pi)^{1/3}$.
- $d < d_H$: coalescence.
- $d > d_H$: breakup[b].

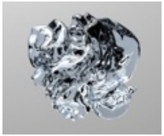
[a] Newman, W., & Sproull, R. (1979). McGraw-Hill, Inc.
[b] Deane, G. B., & Stokes, M. D. (2002). *Nature*, (6900).



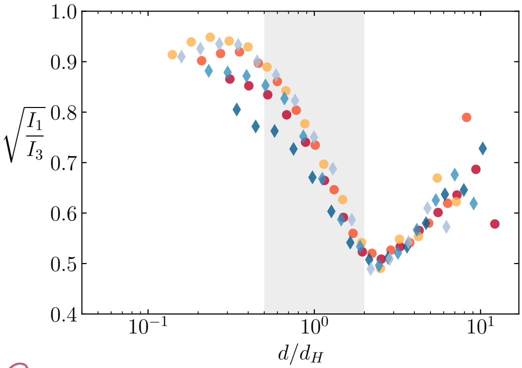

- $d < d_H$: spheroids.
- $d > d_H$: filaments with diameter $d_H$.



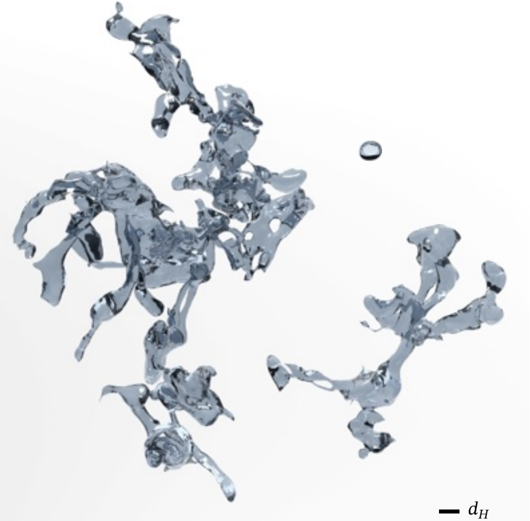



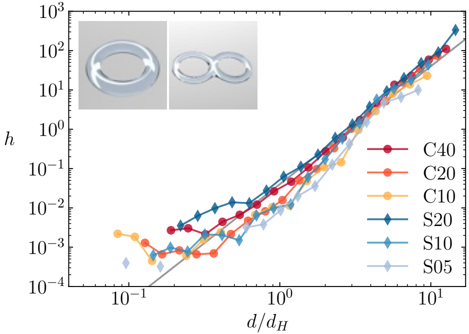
| Aspect ratio $\sqrt{I_1/I_3}$ | 0.946 | 0.633 | 0.905 |
|---|---|---|---|




- $d << d_H$ aspect ratio → 1.
- $d \approx d_H$ droplets are extended.
- $d >> d_H$ aspect ratio → 1.
- This implies the filaments must be coiled.

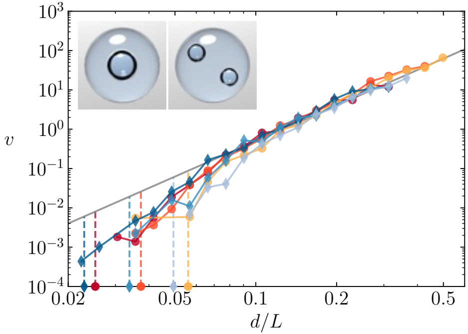



- The large droplets are made up of coiled filaments.
- We see handles and voids.




$$h = 0.04(d/d_H)^3$$

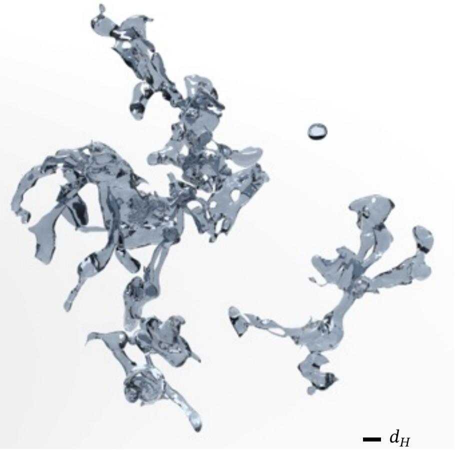
- Constant number of handles per unit length of filament.




$$v = 500(d/L)^3$$

- Constant number of voids per unit volume of droplet.

- Average surface tension can be used to define the Hinze scale $d_H$ in surfactant-laden flows.
- Droplets smaller than $d_H$ are spheroidal.
- Droplets larger than $d_H$ are filamentous.
- Our paper has been accepted for publication in the Journal of Fluid Mechanics[h]




# Thank you!

[j] **Cannon, I.**, Soligo, G., & Rosti, M. E. (2023). *arXiv:2307.15448*

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