Bubble detachment from the sea floor

Ianto Cannon, Stefan Endres, Lutz Mädler, and Marc Avila

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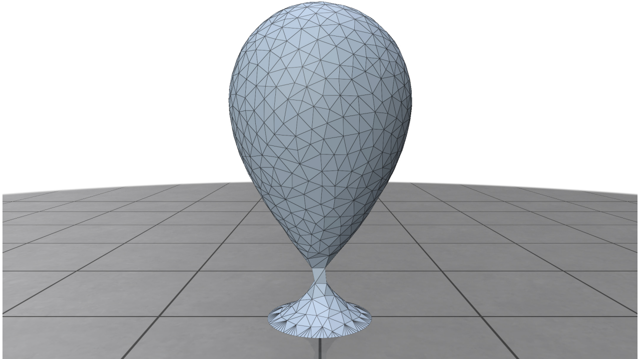
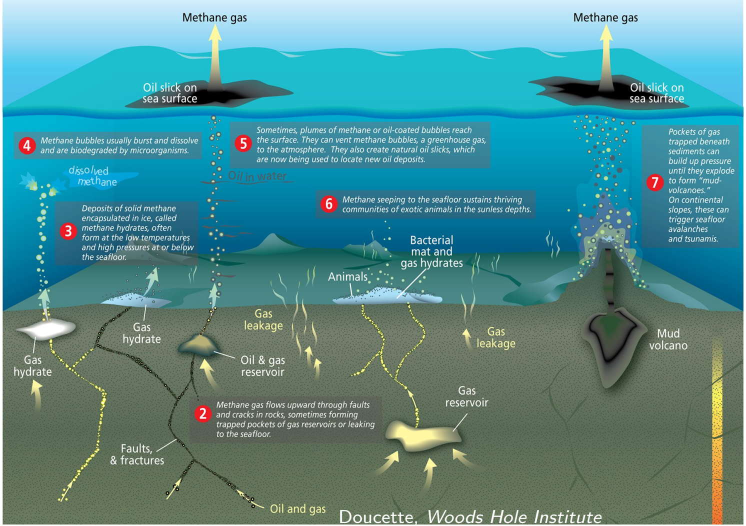


Methane seepage from the seafloor

- 6–12MT/year [Weber, Wiseman, and Kock, *Nat. Commun.* 2019]
- From anoxic sediments, geological reservoirs and degrading methane hydrate deposits
- Bubbles with radius $\lesssim$ 3mm dissolve and biodegrade [McGinnis et al., *J. Geophys. Res. Oceans* 2006].
- Methane is a greenhouse gas



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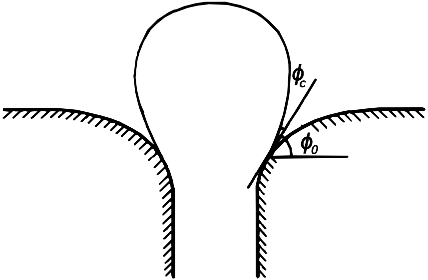
Aims

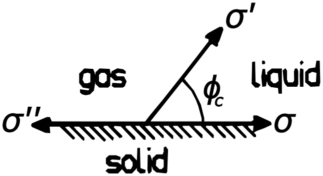
Question

What size are the bubbles when they detach from the sea floor?

Bubble formation

- Gas is emitted from pores in the sea floor.
- The contact angle $0 < \phi_c < \pi$ depends on the affinities of the three phases.

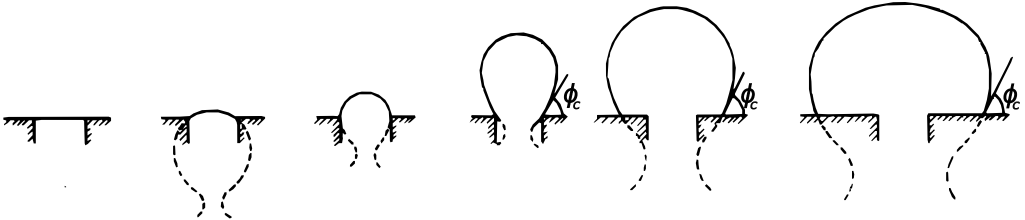




Chesters, *Int. J. Multiph. Flow* 1978

Pinned and spreading bubbles

- At first, the contact line lies on the curved edge of the pore as the bubble grows.
- As long as $\phi_0 > \phi_c$, the bubble is **pinned** to the pore.
- When $\phi_0 = \phi_c$, the bubble **spreads** across the seafloor.



Chesters, *Int. J. Multiph. Flow* 1978

Classical detachment models

The capillary length $\lambda \equiv \sqrt{\frac{\sigma}{(\rho_{\mathrm{out}} - \rho_{\mathrm{in}})g}} \approx 2.7mm$.

Pinned bubbles: Tate volume

$$V = 2\pi\lambda^2 r_c \tag{1}$$

- Tate, *Phil. Mag.* 1864
- Balance between buoyancy and surface tension
- Assumes detachment when $\phi_0 = \pi/2$

Spreading bubbles: Fritz volume

$$V = 0.887\,\lambda^3\phi_c^3 \tag{2}$$

- Fritz, *Physik. Zeitschr.* 1935
- Depends on contact angle ϕ_c
- Accurate to leading order in λ

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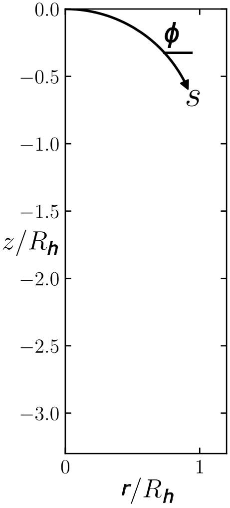
Bubble shape

- The curvature of the interface is given by the Young-Laplace equation,

$$\sigma\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=p_{out}-p_{in} \qquad (3)$$

- Assume axial symmetry (Bashforth and Adams, 1883),

$$\lambda\frac{d\phi}{ds}+\frac{\lambda\sin\phi}{r}=\frac{2\lambda}{R_\mathrm{h}}+\frac{z-h}{\lambda}. \qquad (4)$$



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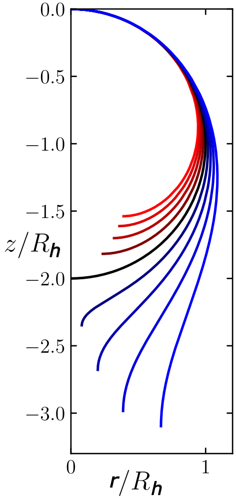
Bubble shape

$$\lambda \frac{d\phi}{ds} + \frac{\lambda \sin\phi}{r} = \frac{2\lambda}{R_{\mathrm{h}}} + \frac{z-h}{\lambda}. \quad (5)$$



- Start from apex and solve using a shooting method.
- Make 10^4 guesses in $10^{-2}\lambda < R_h < 10^2\lambda$.
- Step size $ds = 10^{-4}\lambda$
- Use the step method of Bashforth and Adams, *1883*.
- https://github.com/Leibniz-IWT/ddgclib

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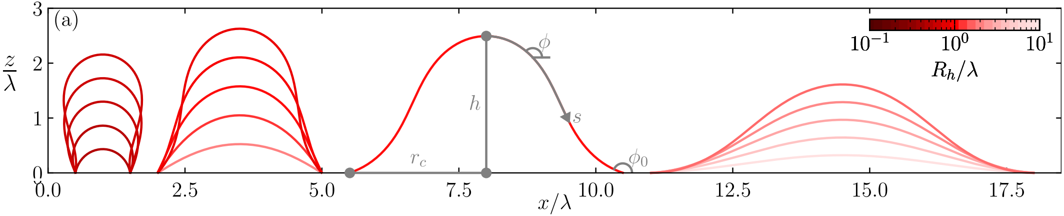
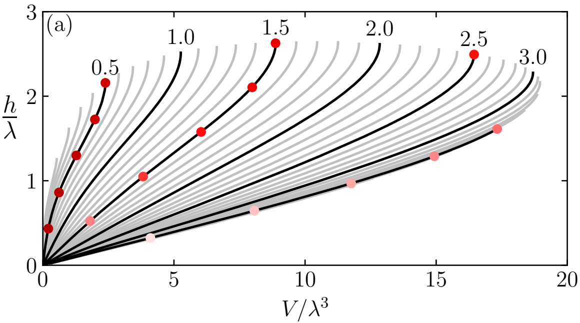
Pinned bubbles

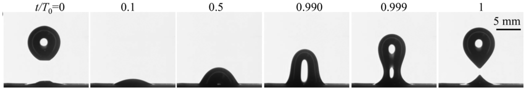
For pinned bubbles, we stop the integration when r reaches the pore radius, $r_c/\lambda \in \{0.5, 1.5, 2.5, 3.5\}$.



Growth of pinned bubbles

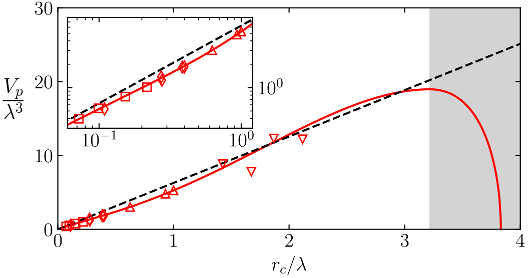


- Volume $V = \int_0^h \pi r^2 \, dz$.
- The bubble begins at the bottom-left.
- The height and volume increase together until:
- The largest volume solution is reached, the height increases without a further increase in volume, and the bubble detaches axisymmetrically.



O'Coin and Ling, *Physics of Fluids* 2024

Detatchment volume of pinned bubbles



- dashed line: $V = 2\pi\lambda^2 r_c$ [Tate, *Phil. Mag.* 1864]
- deviations arise due to the non-spherical interface shape under buoyancy
- experimental measurements:
  1. squares: bubbles emitted from tubes in water: Lesage and Marois, *Int. J. Heat Mass Transf.* 2013
  2. diamonds: bubbles in carbonated water from millimetric cavities: Mori and Baines, *Int. J. Heat Mass Transf.* 2001,
  3. triangles: pendant drops: Gunde et al., *J. Colloid Interface Sci* 2001.

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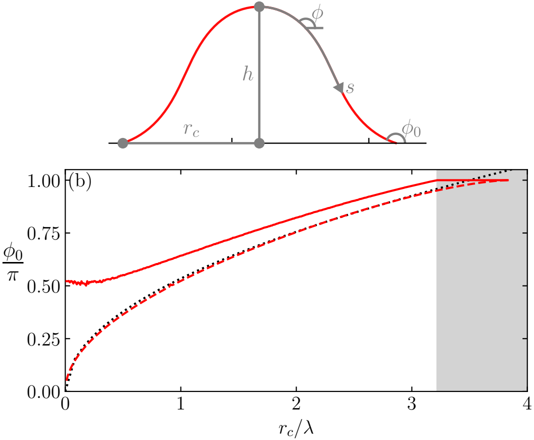
Foot angle of pinned bubbles



- For cavities with $r_c > 3.219\,\lambda$, ϕ_0 reaches π and the interface becomes unstable to non-axisymmetric perturbations Michael and Williams, *Proc. R. Soc. Lond., A* 1976.
- No stable bubble exists for $r_0 > 3.832\,\lambda$ Plateau, *1873*.
- Dashed red line: minimum contact angle during bubble evolution.
- If the surface contact angle ϕ_c exceeds this minimum, the bubble will spread across the surface.
- Dotted black line is the fit $\phi_0/\pi = \sqrt{r_c/3.5\lambda}$.

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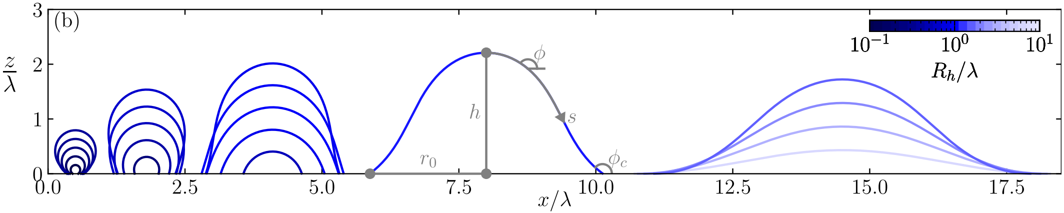
Spreading bubbles

For spreading bubbles, we stop the integration when ϕ reaches the contact angle $\phi_c/\pi \in \{0.2, 0.4, 0.6, 0.8, 1.0\}$.



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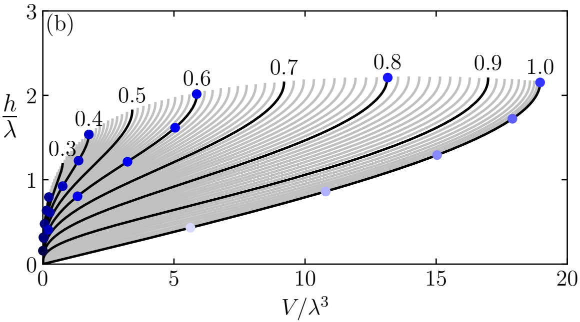
Growth of spreading bubbles



- At top of each ϕ_c curve, the height increases without a further increase in volume, and the bubble detaches axisymmetrically.
- Greater the contact angle ϕ_c, the larger the detachment volume V.
- Tallest possible bubble: $h = 2.642\,\lambda$

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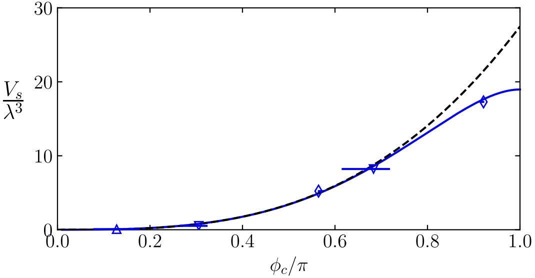
Detatchment volume of spreading bubbles



- For small contact angles, the Fritz prediction is accurate.
- As $\phi_c \to \pi$, however, the detachment volume saturates below Fritz's extrapolation at a maximum of $V = 18.96\,\lambda^3$.
- experimental measurements:
  1. up-triangles: water electrolysis: Demirkır et al., *Langmuir* 2024
  2. down-triangles: boiling: Allred, Weibel, and Garimella, *Int. J. Heat Mass Transf.* 2021
  3. diamonds: air bubbles over superhydrophobic surfaces: Huang and Li, *Phys. Rev. Fluids* 2026.

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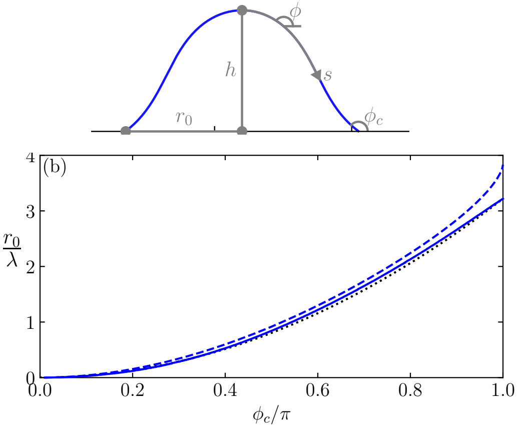
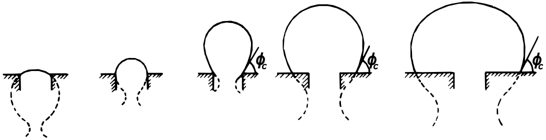
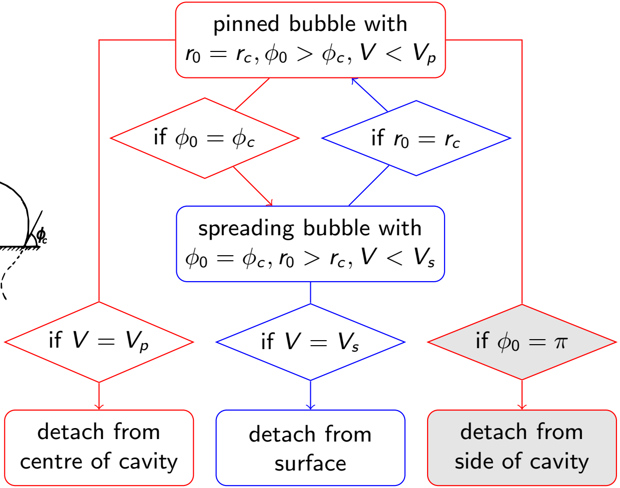
Foot radius of spreading bubbles



- solid line: radius of the bubble foot at departure
- black dotted line: fit $r_0 = 3.219\lambda(\phi_c/\pi)^2$.
- dashed line: maximum foot radius during evolution
- the contact line initially spreads and then partially recedes prior to detachment.

Decision tree





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Ocean Implications

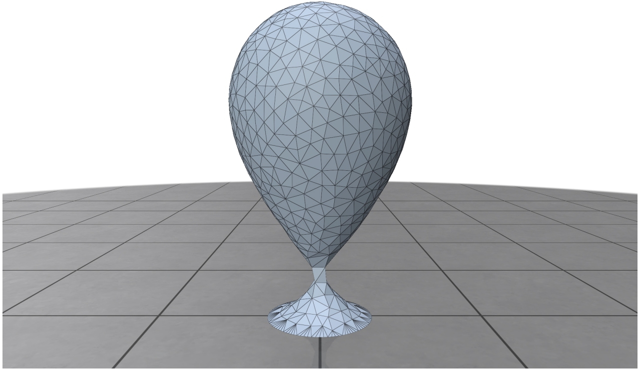
Significance	ϕ_0/π	r_0/λ	h/λ	R_h/λ	V/λ^3	Radius/mm
Largest pinned bubble	1.000	3.219	2.152	1.274	**18.96**	**4.46**
Largest spreading bubble	1.000	3.219	2.152	1.274	**18.96**	**4.46**
Tallest pinned bubble	0.7750	1.720	**2.642**	0.8867	10.62	3.68
Tallest spreading bubble	0.8330	2.297	**2.213**	1.126	14.48	4.08
Largest stable foot radius	1.000	**3.832**	0.000	∞	0.000	0.00

- The largest bubbles have a radius $>$ 3mm, and can reach the sea surface.
- The pore size and contact angle are critical for determining whether bubbles will reach the atmosphere.
- Direct impact on climate.

Aims

Question

Can we predict the shape of generic 3D bubbles?



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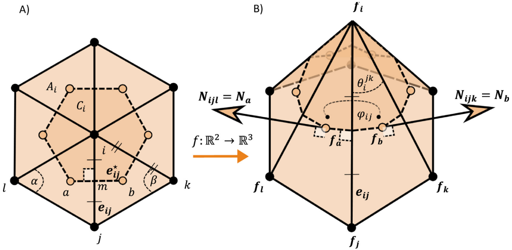
Surface tension force

- Triangulate the interface so that vertices lie on the surface.
- Compute the total curvature at each vertex using discrete differential geometry.

$$\widehat{H\mathbf{N}A_i} = \frac{1}{2} \sum_{j \in st(\mathbf{f_i})} \left(\cot \alpha_{ij} + \cot \beta_{ij}\right) \left(\mathbf{f}_i - \mathbf{f_j}\right). \tag{6}$$

- Apply a surface tension force $\mathbf{S_i} = \sigma \widehat{H\mathbf{N}A_i}$.
- Endres, Avila, and Lutz, *Chem. Eng. Sci.* 2022



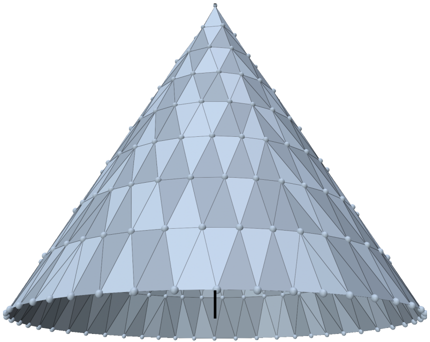
Pressure forces

- Initialise the bubble as a cone with volume V_0 and base radius R_{foot}
- Outer pressure assumed to be hydrostatic.

$$p_{\text{out}} = \Delta\rho g z \tag{7}$$

- Inner pressure enforces volume conservation.

$$p_{\text{in}} = \frac{2\sigma}{R}\left(\frac{V_0}{V}\right)^5 \tag{8}$$



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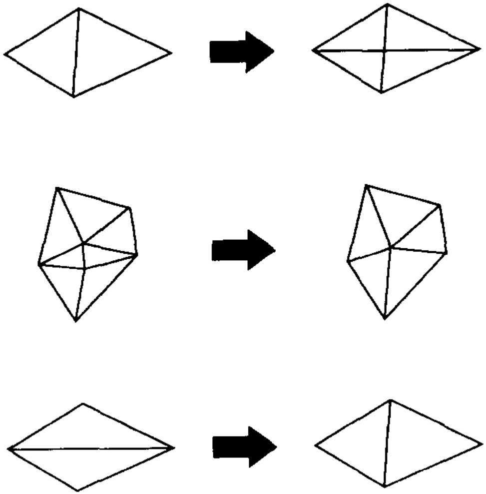
Remeshing

Use the method described in Unverdi and Tryggvason, *J. Comp. Phys.* 1992;

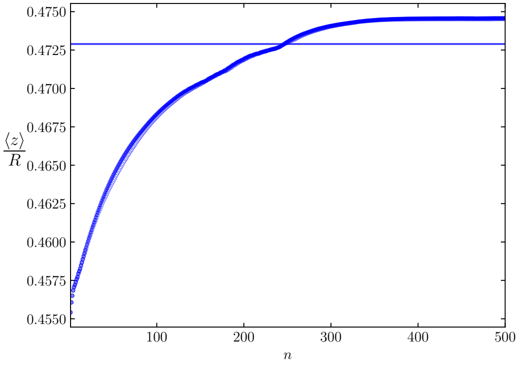
- Split edges longer than $0.2R$
- Merge edges shorter than $0.1R$
- Reconnect diagonals if it shortens them by more than a factor of 1.5.



Minimising free energy

Use the Adams Bashforth method to find the locations $\mathbf{x_i}$ where resultant forces $\mathbf{F_i}$ on the vertices are zero;

$$\mathbf{x_i^{n+1}} = \mathbf{x_i^n} + \alpha \frac{3}{2} \mathbf{F_i^n} - \alpha \frac{1}{2} \mathbf{F_i^{n-1}} \tag{9}$$



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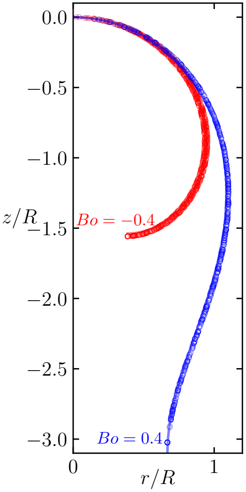
Validation: bubble shape

For a fixed contact line, we can replicate the bubble shape for a range of Bond numbers, $\mathrm{Bo} = \dfrac{\Delta \rho g R^2}{\sigma}$.



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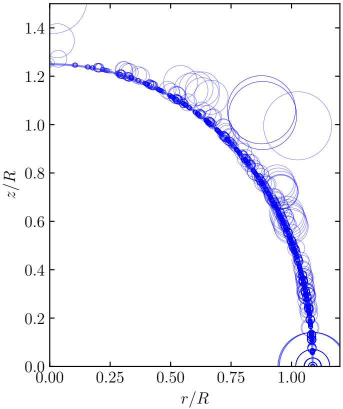
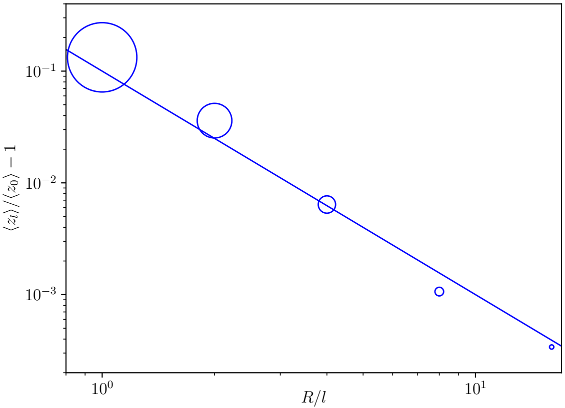
Validation: grid dependence





The error in the height of the centre of mass has a quadratic dependence on the grid size,

$$\frac{\langle z_l \rangle}{\langle z_0 \rangle} - 1 = 0.1 \left(\frac{R}{l} \right)^{-2}. \tag{10}$$

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Conclusions

We provide universal laws for bubble detachment from surfaces.

- Predicted bubble size and detachment mode for each pore radius and contact angle.
- Developed a 3D formulation with second-order convergence.

Applies to:

- methane seepage
- electrolysis
- boiling
- pendant drops.

Thank you!

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