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Particles in the ABC flow, and morphology of droplets in turbulent flows

Ianto Cannon,

Complex Fluids and Flows Unit, Okinawa Institute of Science and Technology

Part one:
Particles in the turbulent ABC flow

Alessandro Chiarini, Ianto Cannon & Marco E. Rosti

Particles in **turbulent flows**

- Particle fluid interaction affects the fluid motion and the paths of the particles
- Modulation of the large-scale flow is relevant to sandstorms in engine cooling/cleaning

Sandstorm

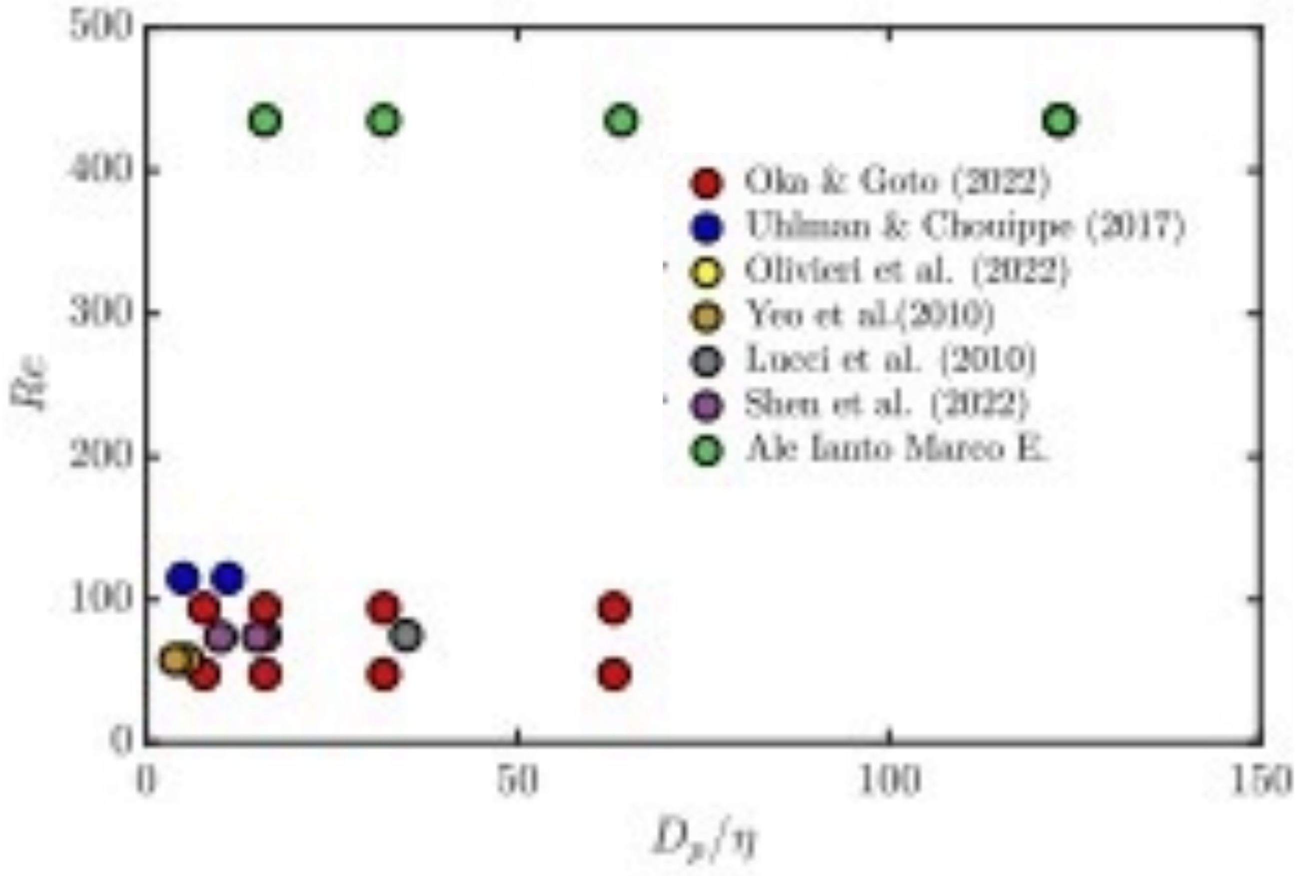
Credit: Barcroft media, 2010

Pollen dispersal

Credit: Robert J. Erwin

Recent work in particle-laden turbulence

- Recent reviews have stated a gap in our understanding between small particles and large massive particles
- We explore intermediate size particles ($\eta < D_p < L$)
- We investigate a high turbulence intensity ($Re_\lambda = 435$)

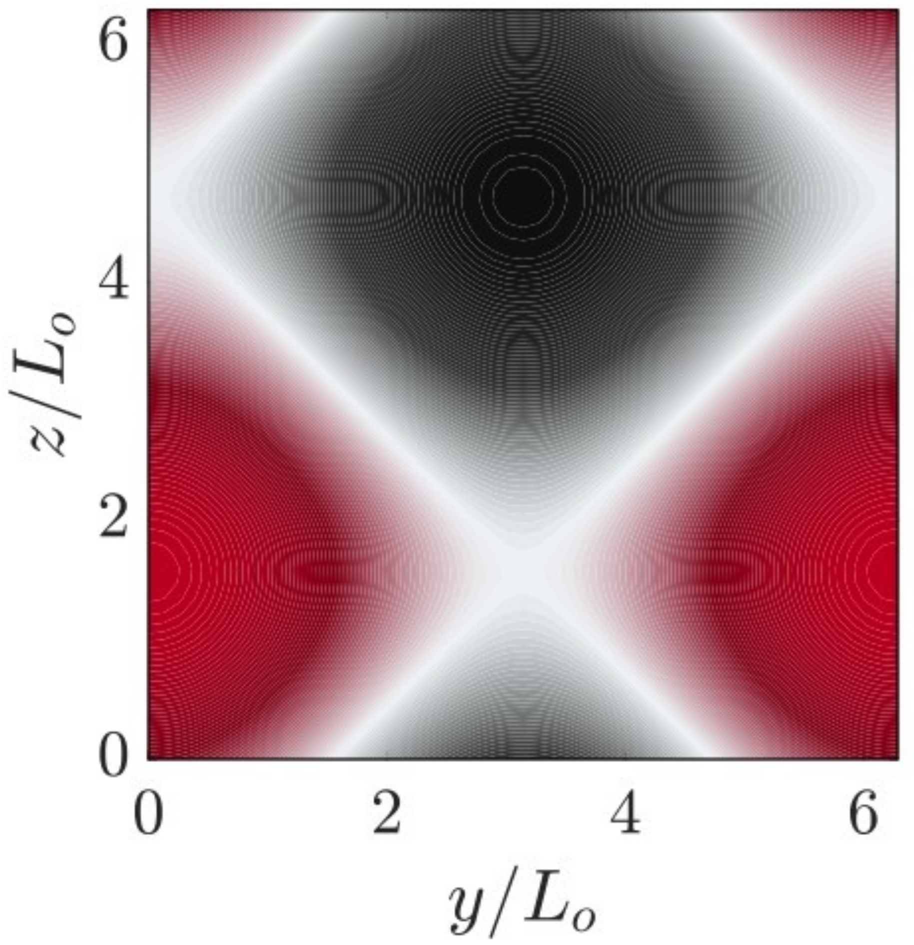


Brandt & Colletti *Ann. Rev. Fluid Mech.* (2021)

Oka & Goto, *J. Fluid Mech.* (2022)

Our goals

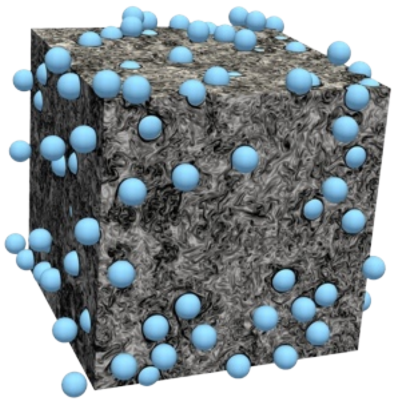
- *How do solid particles modulate the largest scales of the flow?*
- We choose Arnold-Beltrami-Childress (ABC) forcing with $A = B = C = 1$
    - $$\boldsymbol{f} = \left(\frac{2\pi}{L}\right)^2 F_o \begin{pmatrix} A\sin(2\pi z/L) + C\sin(2\pi y/L) \\ B\sin(2\pi x/L) + A\sin(2\pi z/L) \\ C\sin(2\pi y/L) + B\sin(2\pi x/L) \end{pmatrix}$$
    - Triperiodic cellular forcing
    - Rotation symmetry in x, y, z
    - 3D mean shear
    - No need for walls
- *How do particles disperse in the flow?*



Musacchio & Boffetta, *Phys. Rev. E* (2014)

Podvigina, Phys. D: Nonlin. Phenom. (1994)

Numerical method



- 10^4 spherical particles

$$m^p \frac{dU_i^c}{dt} = \oint_{\partial \mathcal{V}^p} \sigma_{ij} n_j \, dA + F_i^{\mathrm{ext}},$$

$$I^p \frac{d\Omega_i^c}{dt} = \oint_{\partial \mathcal{V}^p} \epsilon_{ijk} r_j \sigma_{kl} n_l \, dA + T_i^{\mathrm{ext}}$$

Hori et al, Comput. & Fluids, 2021

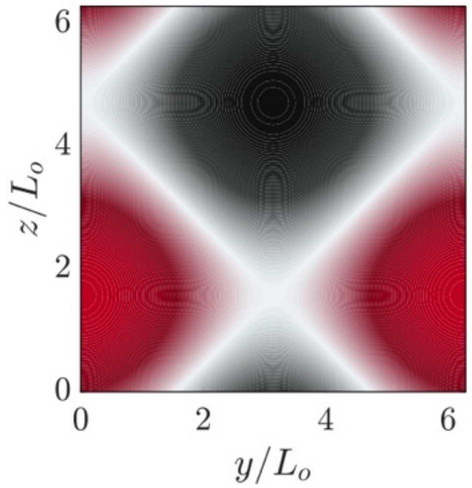
- We solve the flow on a 1024^3 numerical with $Re_\lambda \approx 435$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho_{\mathrm{f}}} \nabla p + \nu \Delta \mathbf{u} + \mathbf{f} \quad + \mathbf{f}_{\mathrm{FS}}$$

$$\nabla \cdot \mathbf{u} = 0$$

Two-way coupled IBM

ABC forcing



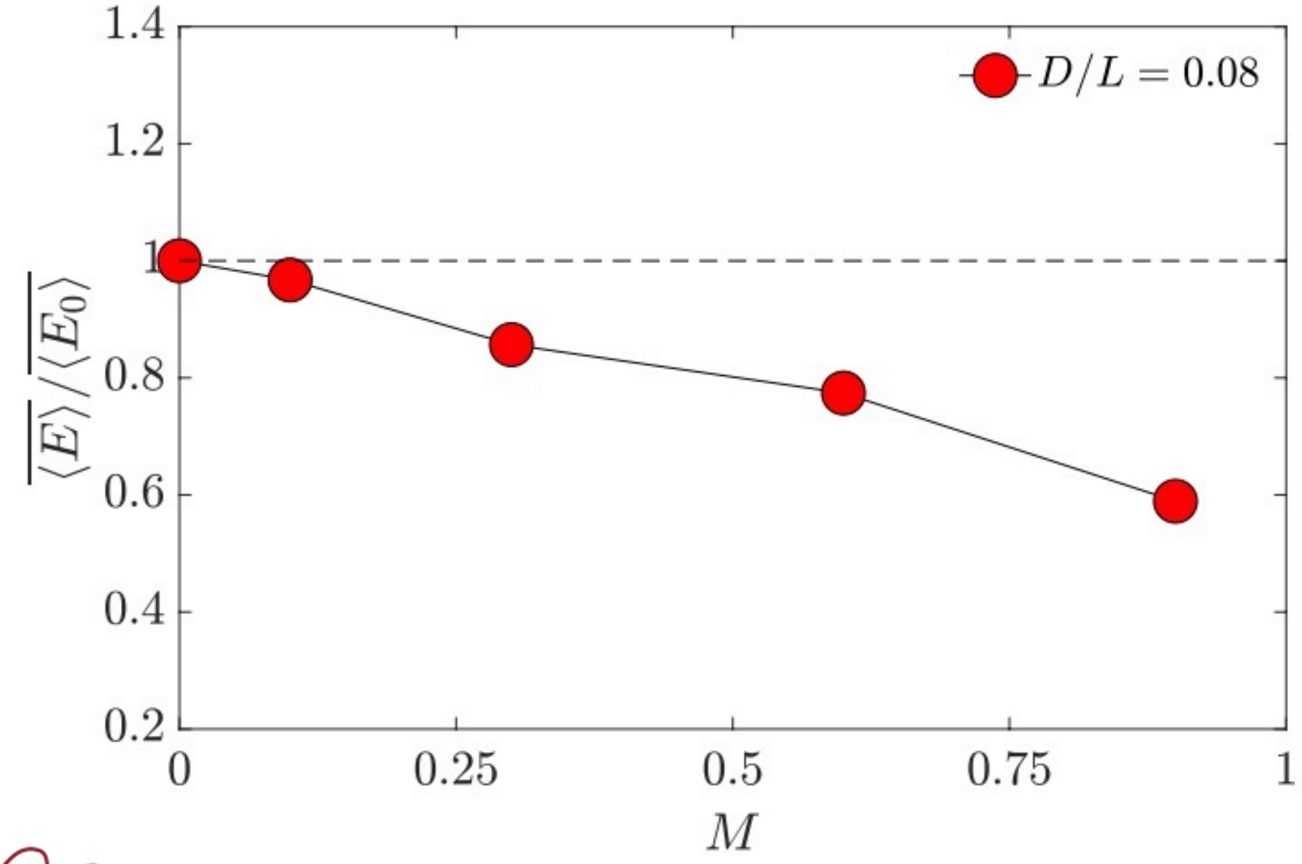
- We vary the particle mass fraction

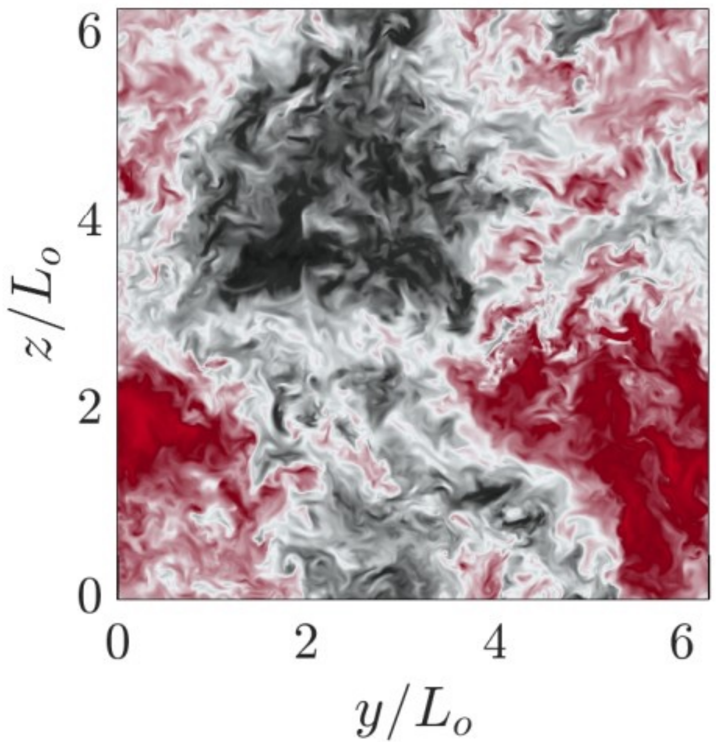
$$M \equiv \frac{m_p}{m_f + m_p}$$

- And the particle diameter D

Kinetic energy modulation

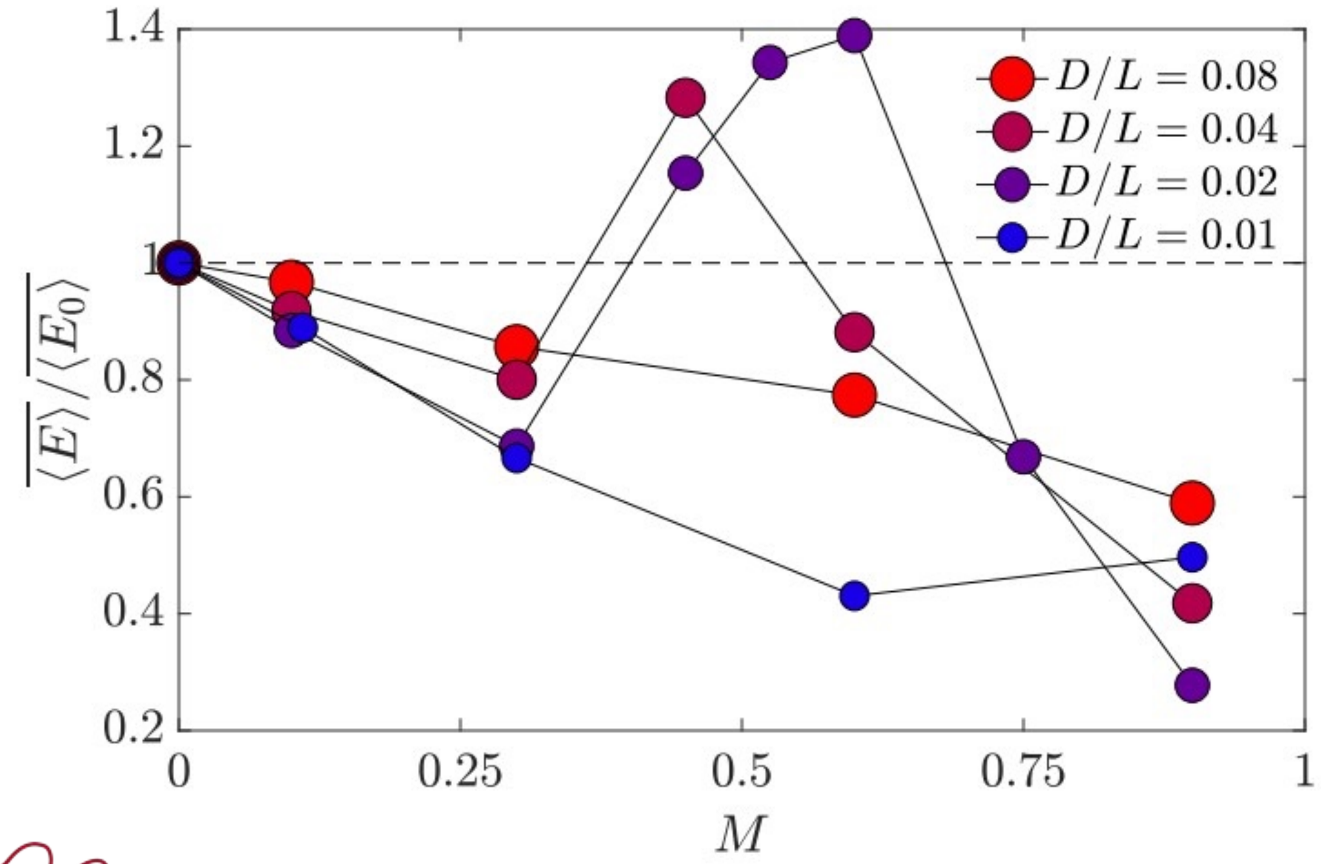
- Generally as particle mass fraction increases, the energy of the flow is suppressed

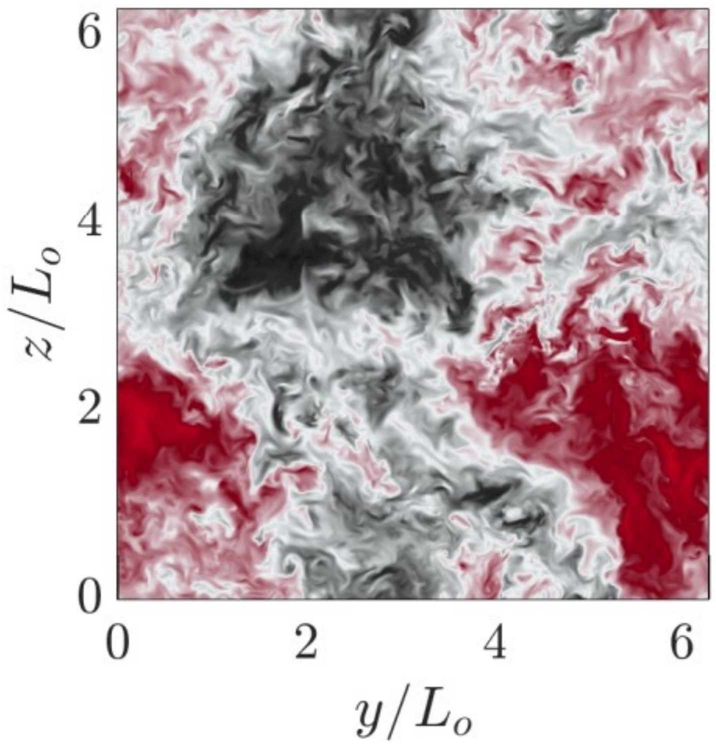




Kinetic energy modulation

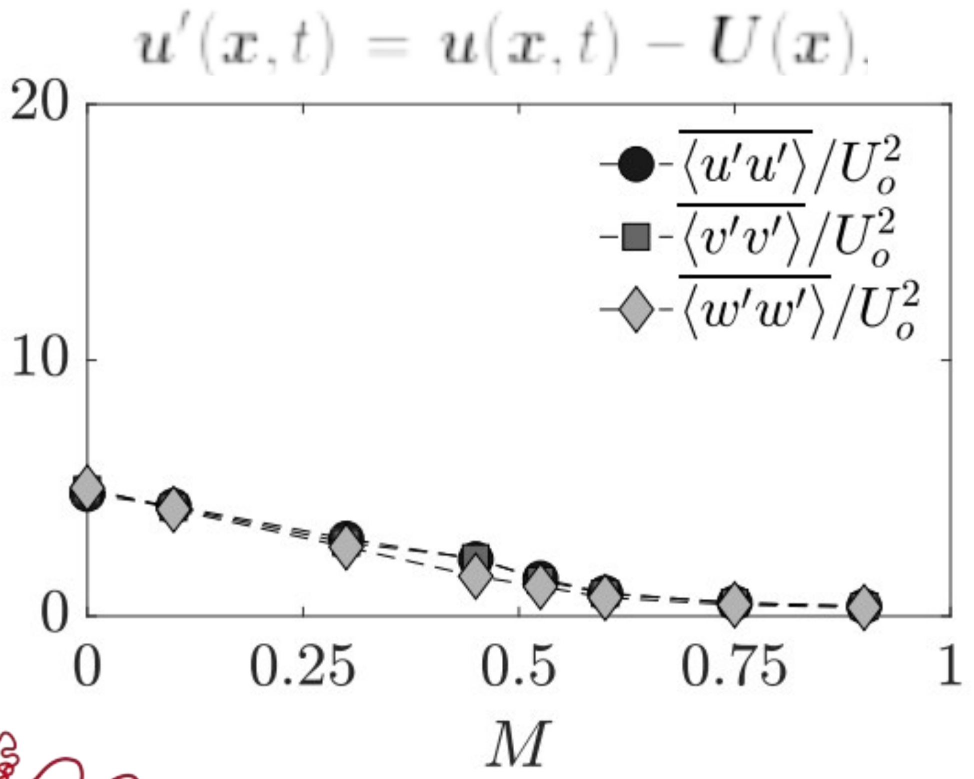
- At some intermediate particle sizes, energy is enhanced!

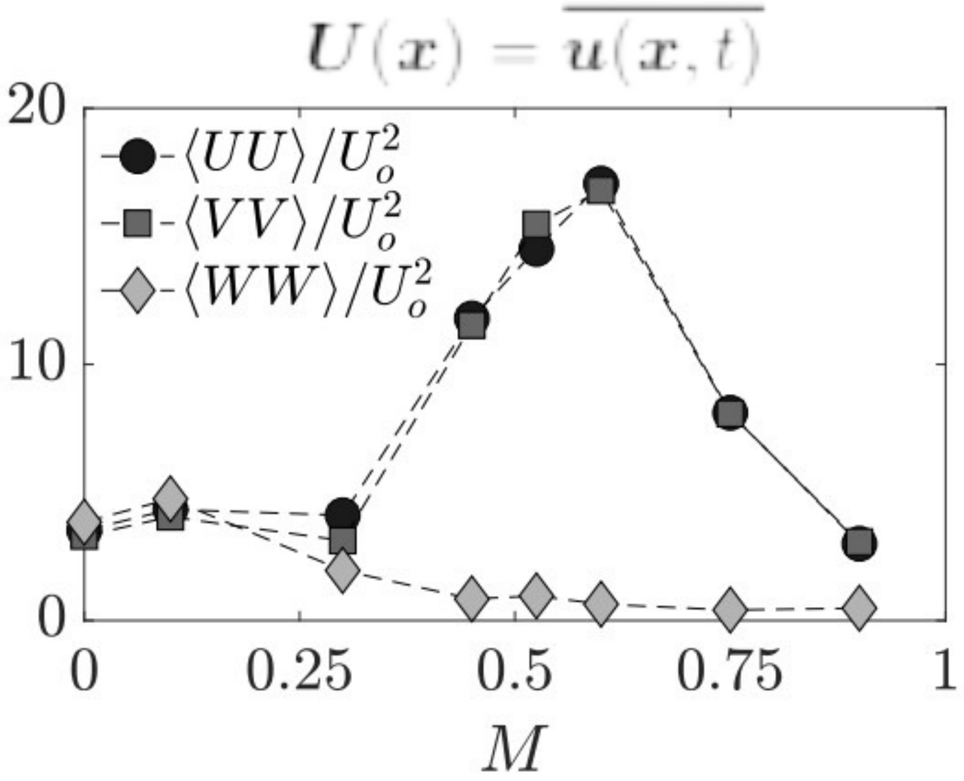




Turbulence modulation

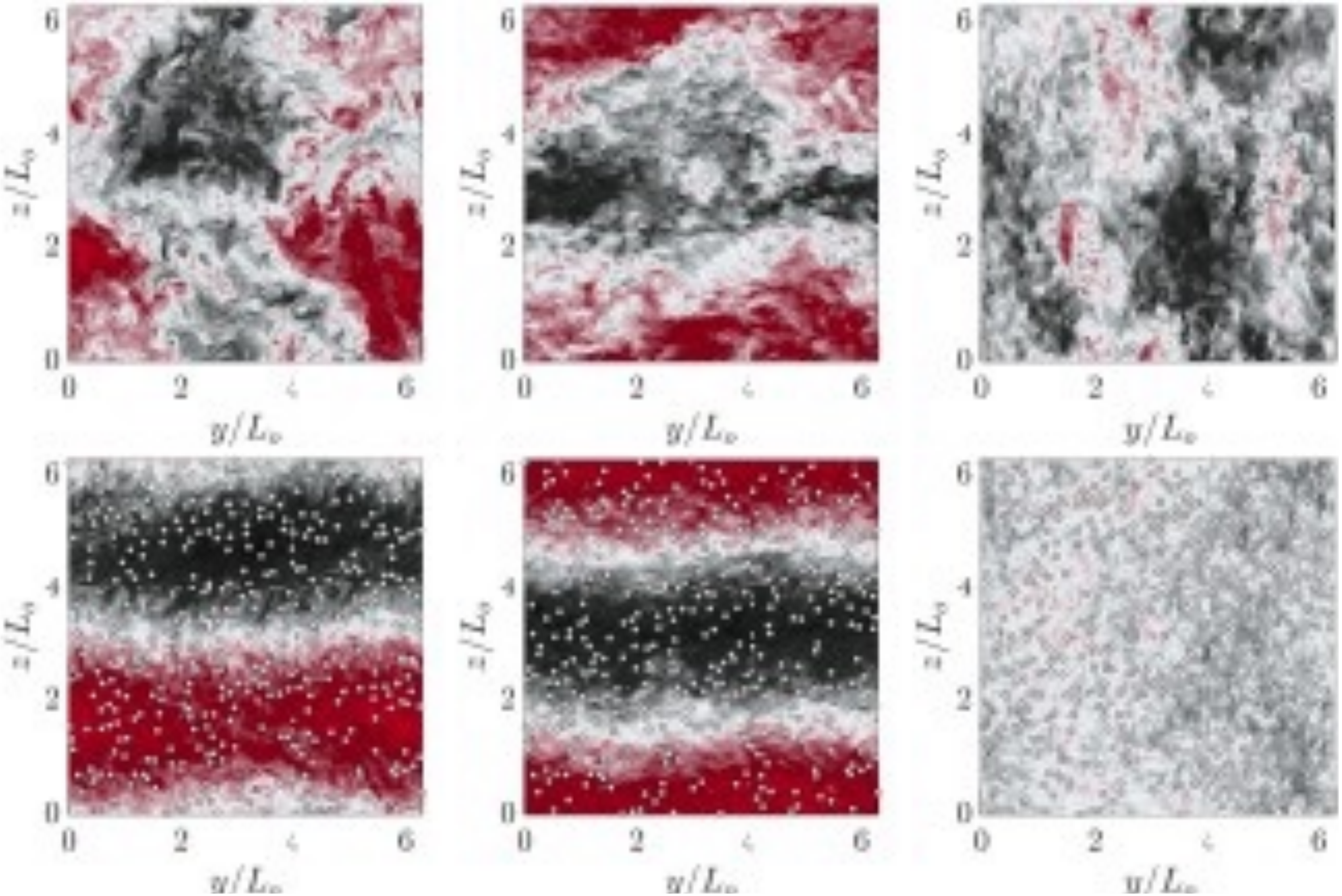
- This is entirely due to an enhancement of the mean (time averaged) flow
- The mean flow is highly anisotropic





Anisotropy of the flow

- The single phase flow resembles the forcing with turbulent fluctuations
- For large particle mass fractions, one direction (z) is suppressed
- the flow becomes almost two-dimensional



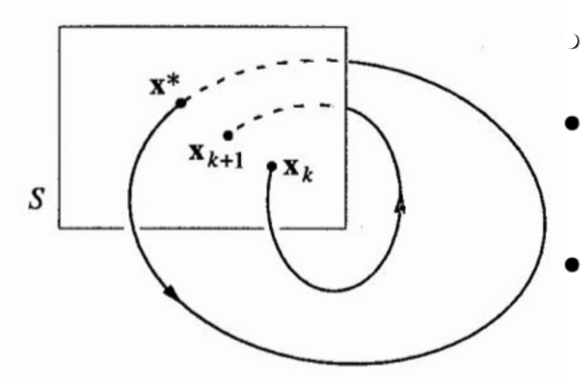
$M = 0$

$M = 0.6$

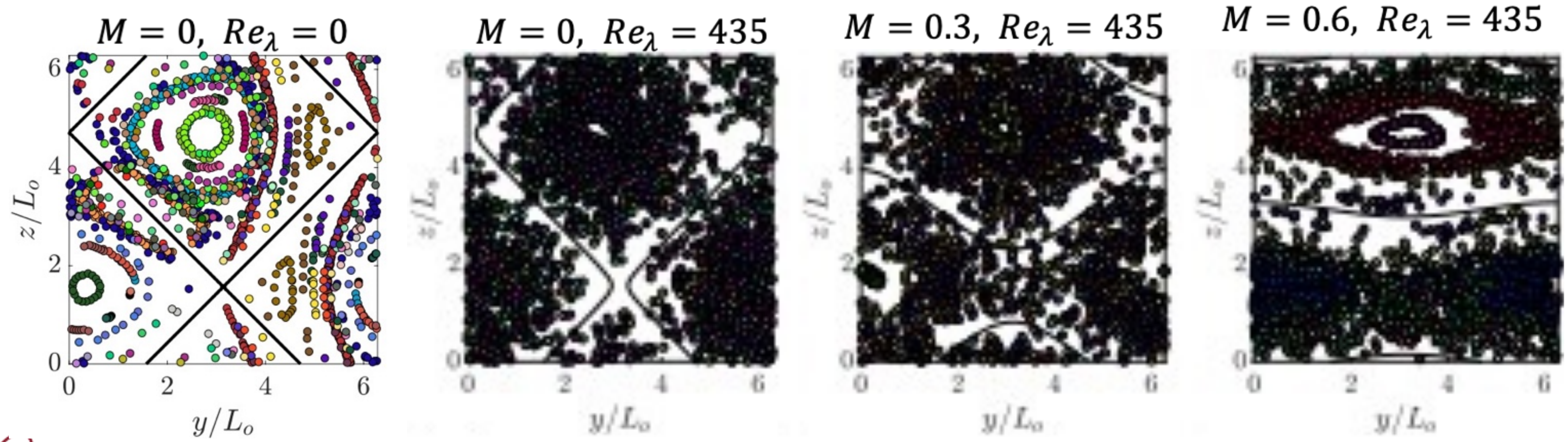
Poincaré, (1892)

Dombre, *J. Fluid Mech.* (1986)

Poincaré maps

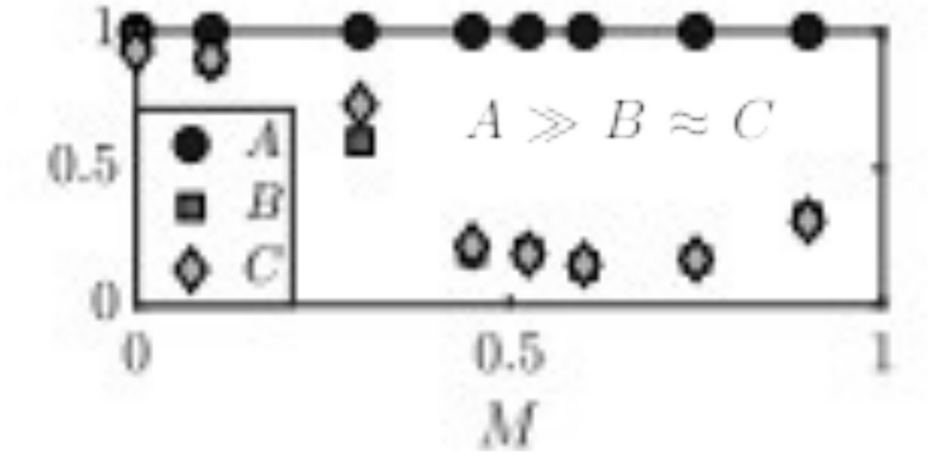


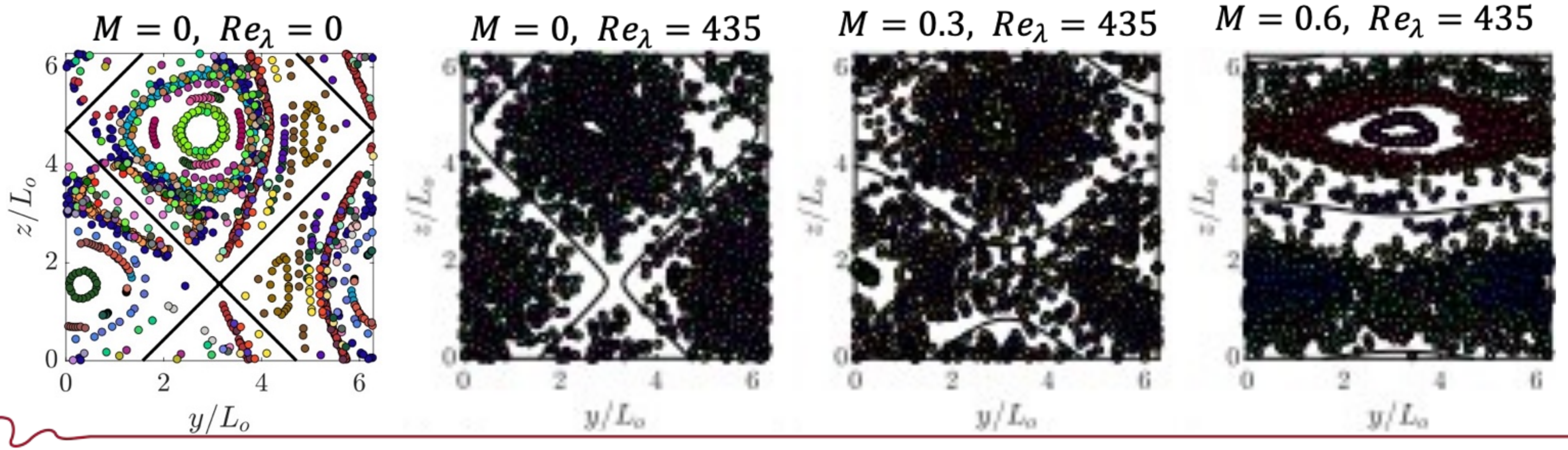
- We make Poincaré maps of the mean flow in the $x = L/2$ plane.
- This is done by following streamlines and marking successive intersections of the plane
- For M=0.3 and M=0.6, the spread of points in the z direction is reduced



Shape of the mean flow

- We calculate the coefficients A,B and C
- For large mass fractions, a new type of ABC flow is seen with B and C reduced.
- We show the loci where the new ABC flow has $u = 0$.
- These coincide with low density regions in the Poincaré maps.



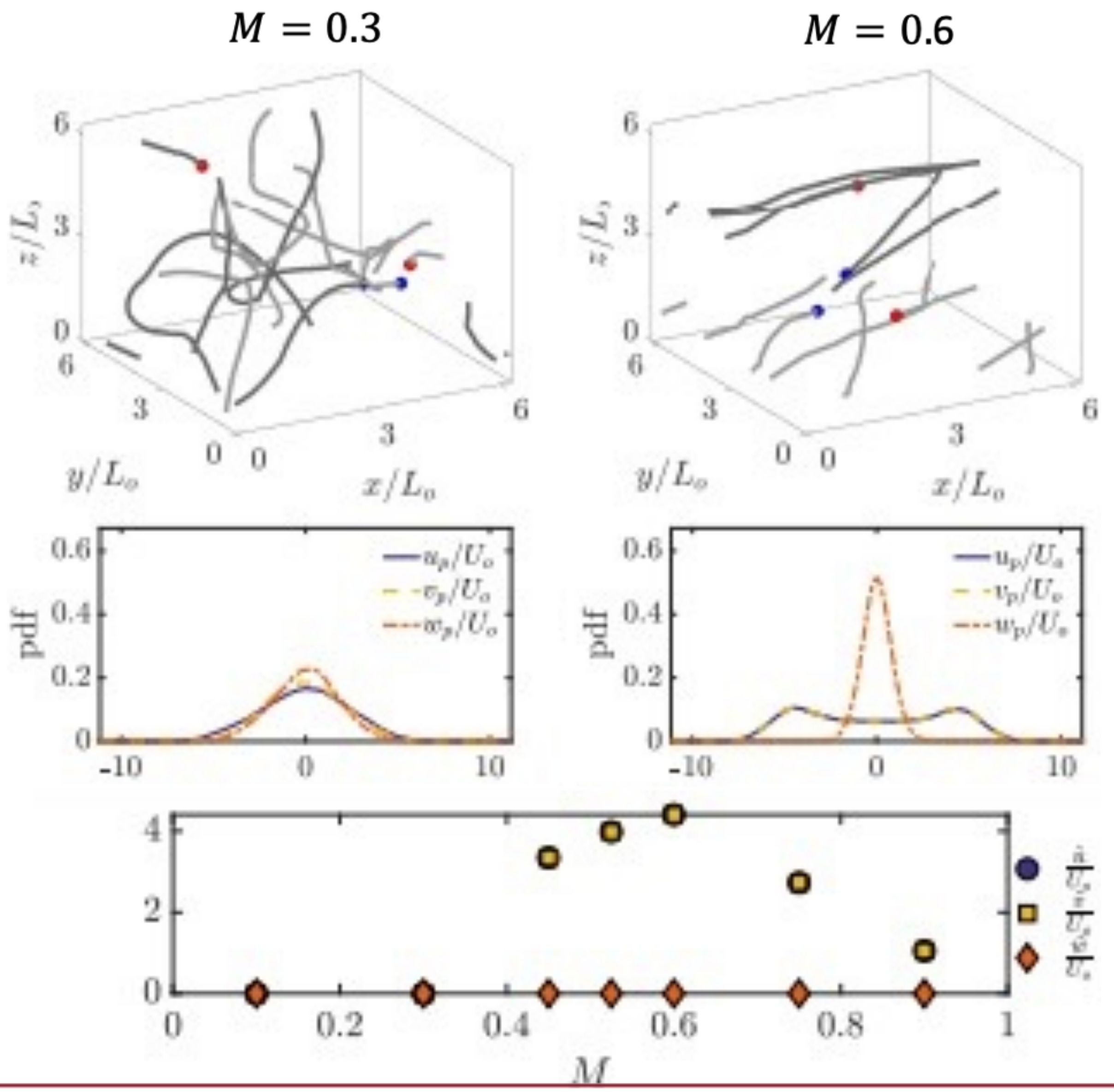


Particle trajectories

- Low mass particles follow the chaotic flow
- The three particle velocity components have the same probability distribution.

- Massive particles move in straight lines
- The z-velocity is strongly peaked at zero, whereas x and y-velocities become bi-modal
- Particles continuously force the flow towards the 2D state

- When $M > 0.3$, the modes depart from zero



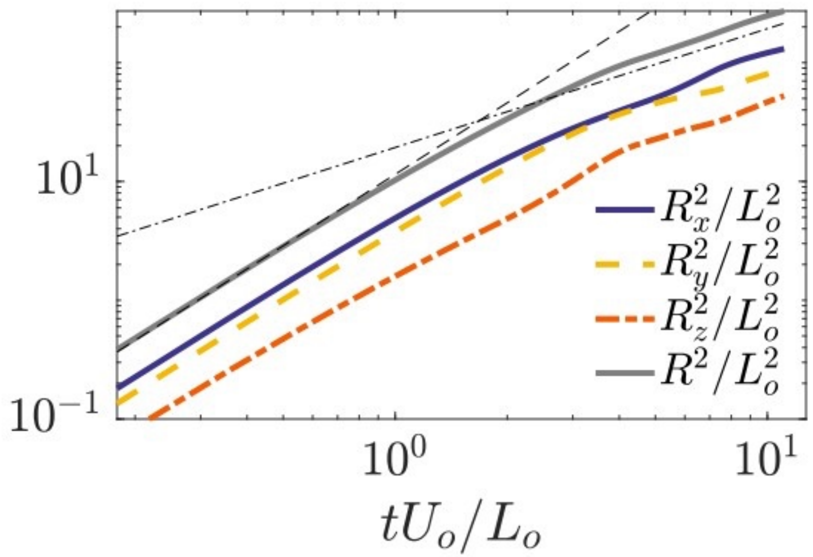
Particle displacement

- The mean-squared displacement of the particles is a measure of the distance travelled after a time t

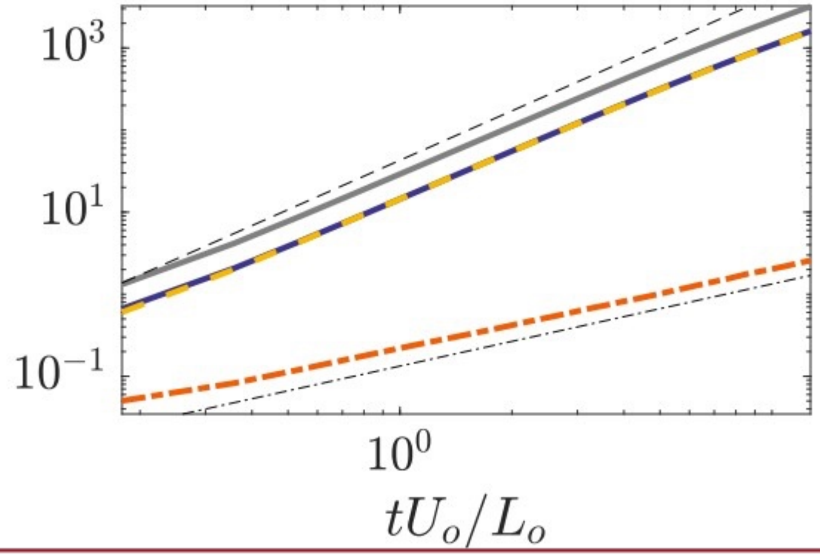
$$R(t)^2 = \langle |\boldsymbol{x}_p(t) - \boldsymbol{x}_p(0)|^2 \rangle_p \sim t^\alpha$$

- The low mass particles have
  - ballistic motion $\alpha \approx 1$
  - followed by diffusive motion $\alpha \approx 2$

- The massive particles have
  - ballistic motion in x and y for all times
  - diffusive motion in z for all time
  - this is a type of anomalous diffusion



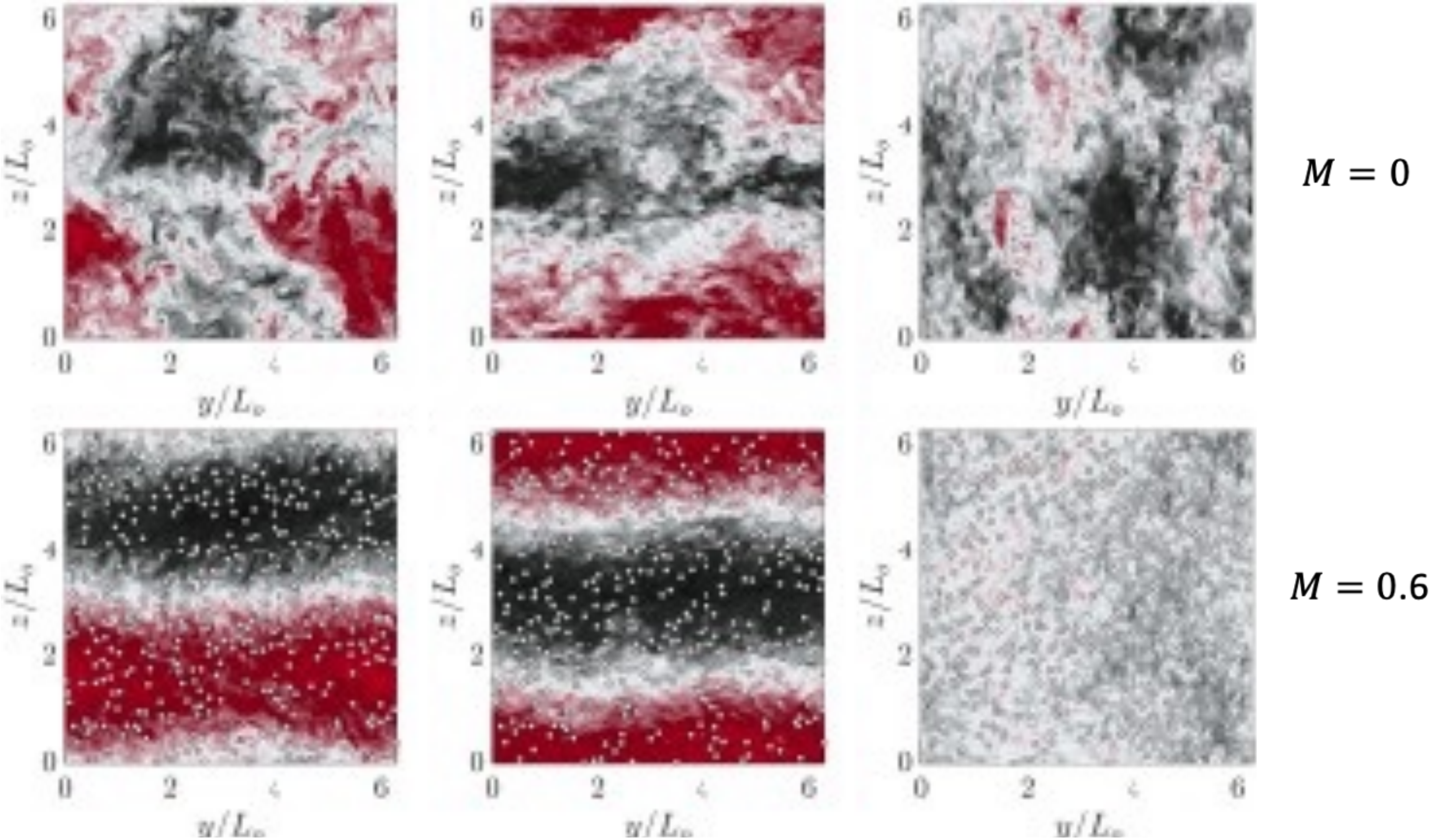
$M = 0.3$

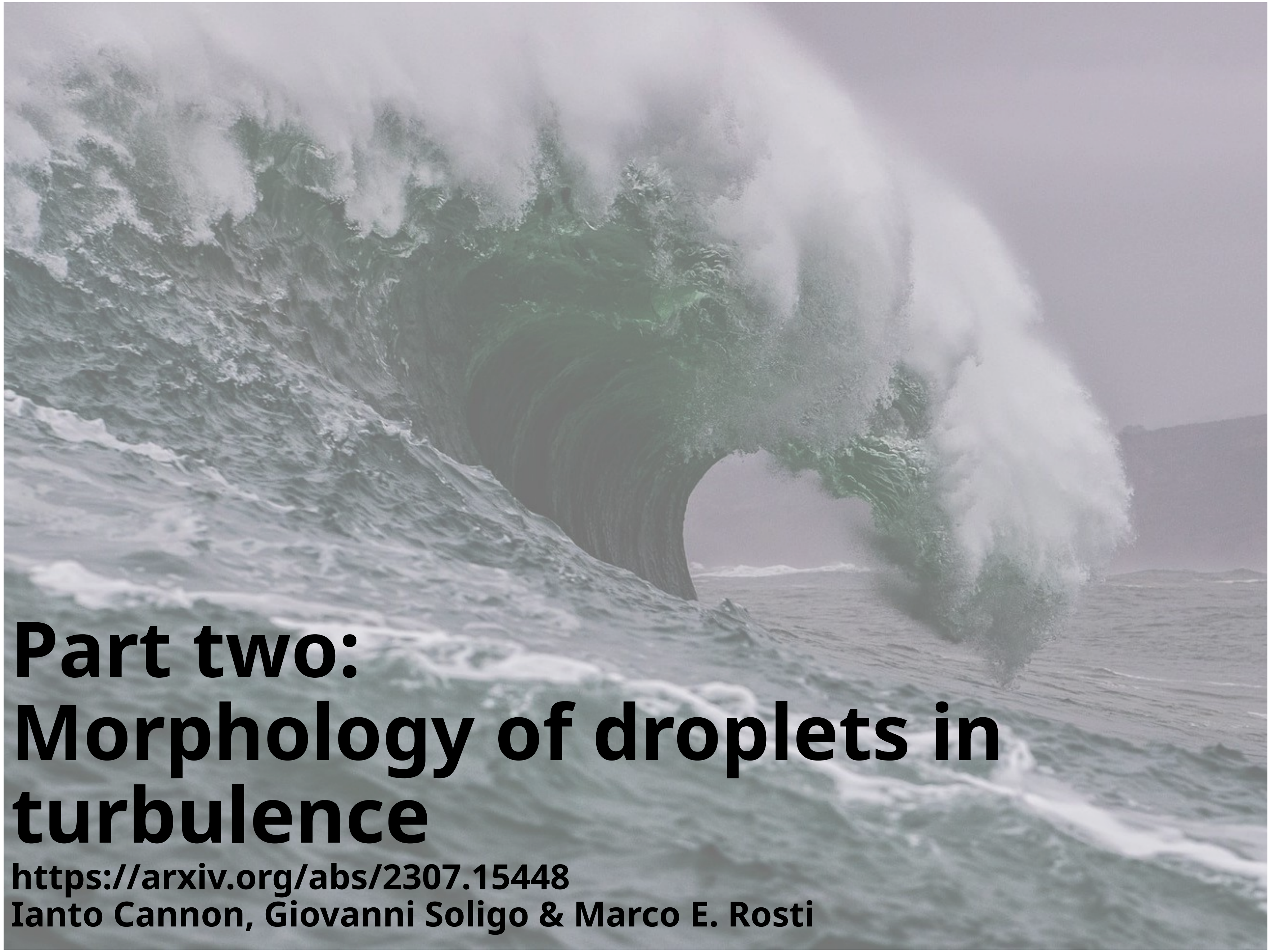


$M = 0.6$

Part 1 conclusion

- The kinetic energy is enhanced at large particle mass fractions
- The flow becomes anisotropic
- This shows the potential to use particles to modify the large scale structures of the flow
- The particles exhibit anomalous diffusion
- Future work:
- Can particles reduce recirculating regions?





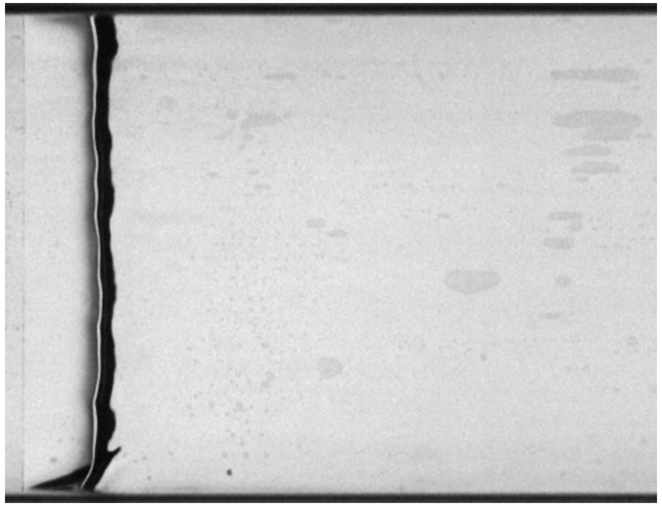
Part two:
Morphology of droplets in turbulence

https://arxiv.org/abs/2307.15448

Ianto Cannon, Giovanni Soligo & Marco E. Rosti

Droplet breakup in nature

Droplet breakup during cough



Kant et al. *Phys. Rev. Fluids* 2023 Bag-mediated film atomization in a cough machine

CO_2 absorption by the ocean

Credit: Linda Gietka

What is the morphology of droplets in turbulence?

- Surfactants are *surface active agents*, compounds that decrease surface tension at an interface
- Eg., soap, lipids in the lungs, salts, and many impurities

How do surfactants affect morphology?

Numerical method

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla \cdot \left[\mu \left(\nabla \mathbf{u} + \nabla \mathbf{u}^T \right) \right] + \nabla \cdot (\tau_c f_\sigma) + \mathbf{f}_s,$$

-fluid motion $Re_\lambda = 180$

$$\nabla \cdot \mathbf{u} = 0,$$

-incompressibility

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\mathbf{u} H) = 0,$$

-volume-of-fluid field

$$\frac{\partial \psi}{\partial t} + \mathbf{u} \cdot \nabla \psi = \nabla \cdot (M_\psi \nabla \mu_\psi),$$

-surfactant concentration

$$f_\sigma = \max[\sigma_{min}, \sigma_0(1+\beta_s \log(1-\psi))]$$

-surface tension

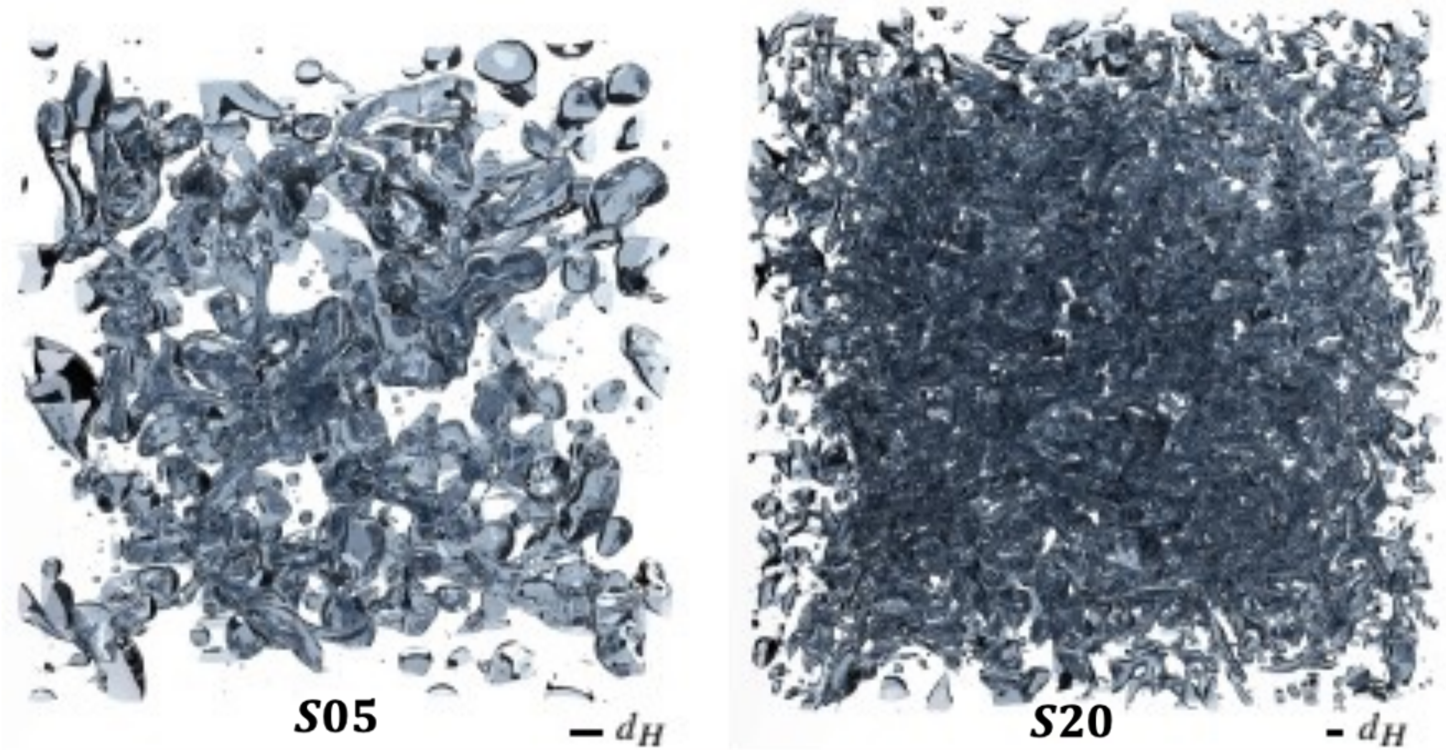
Engblom et *al*., *Comm. Comput. Phys.* (2013)
Yun et *al*., *Appl. Math. Comput.* (2014)
Soligo et *al*., *J. Comput. Phys.* (2019)

Simulation setup

- Statistically-steady turbulence in a triperiodic box with 500^3 grid points
- We define a **Kolmogorov-Hinze diameter** in the presence of surfactant

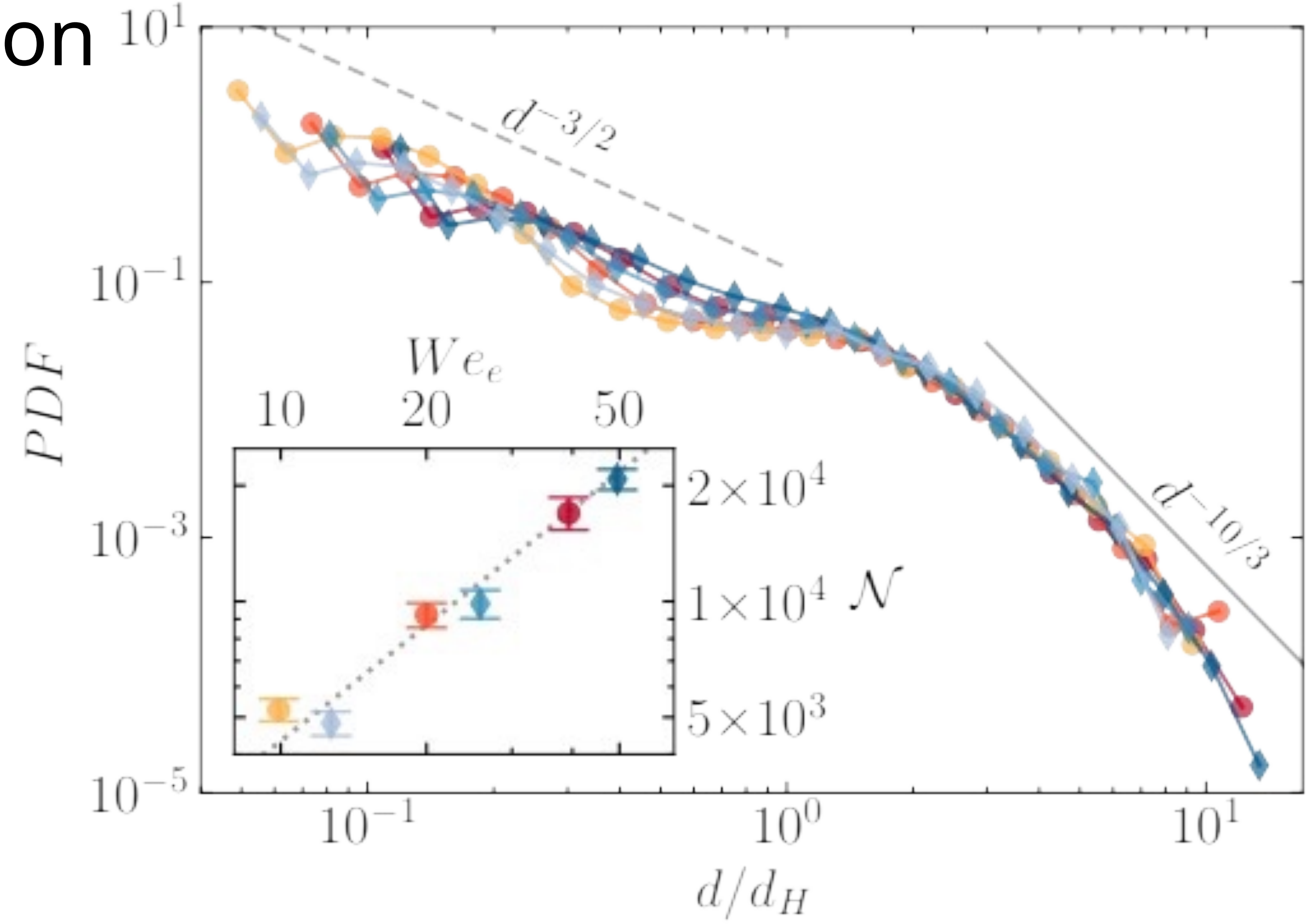
$$d_H \equiv 0.725\langle\sigma\rangle_I^{3/5}\rho^{-3/5}\varepsilon^{-2/5}$$

case		$\langle\phi\rangle$	$\langle\psi\rangle$	We	$d_{H\sigma}/L$
SP	●	0	0	-	-
C10	●	0.1	0	10	**0.0565**
C20	●	0.1	0	20	**0.0392**
C40	●	0.1	0	40	**0.0290**
S05	♦	0.1	0.1	5	**0.0370**
S10	♦	0.1	0.1	10	**0.0317**
S20	♦	0.1	0.1	20	**0.0241**



Droplet size distribution

- We define an equivalent diameter for each droplet, $d \equiv (6V/\pi)^{1/3}$,
- The DSD shows two distinct scalings, which are known to result from coalescence and breakup



Garrett *et al.*, *J. Phys. Oceanograph.* (2000)
Dean, Stokes, *Nature* (2002)

Droplet deformation

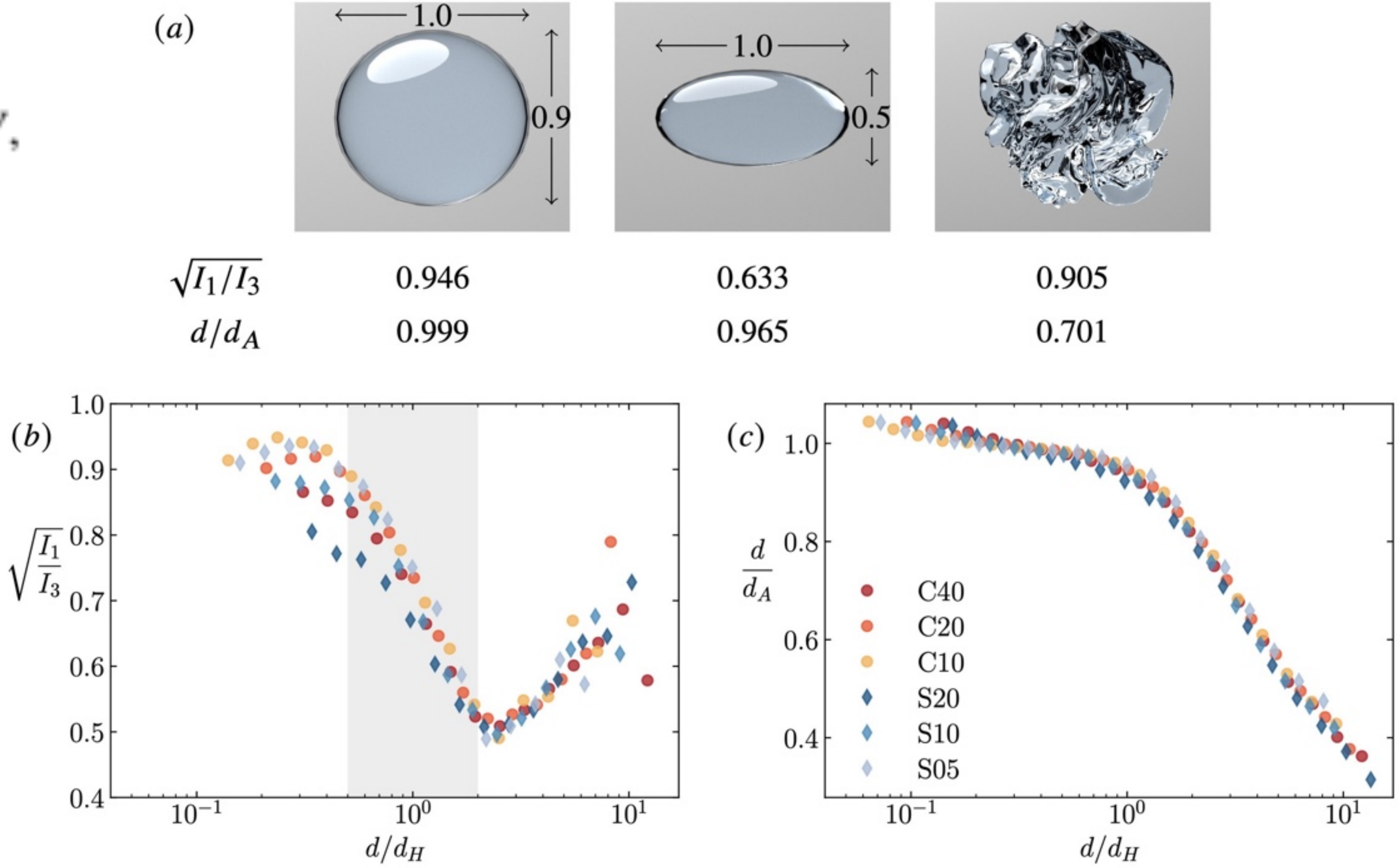
- We measure the aspect ratio

$$\sqrt{I_1/I_3}, \quad \mathbf{I} \equiv \int_V \rho(||\mathbf{r}||^2 \mathcal{I} - \mathbf{r} \otimes \mathbf{r}) dV,$$

- and the sphericity d/d_A, where

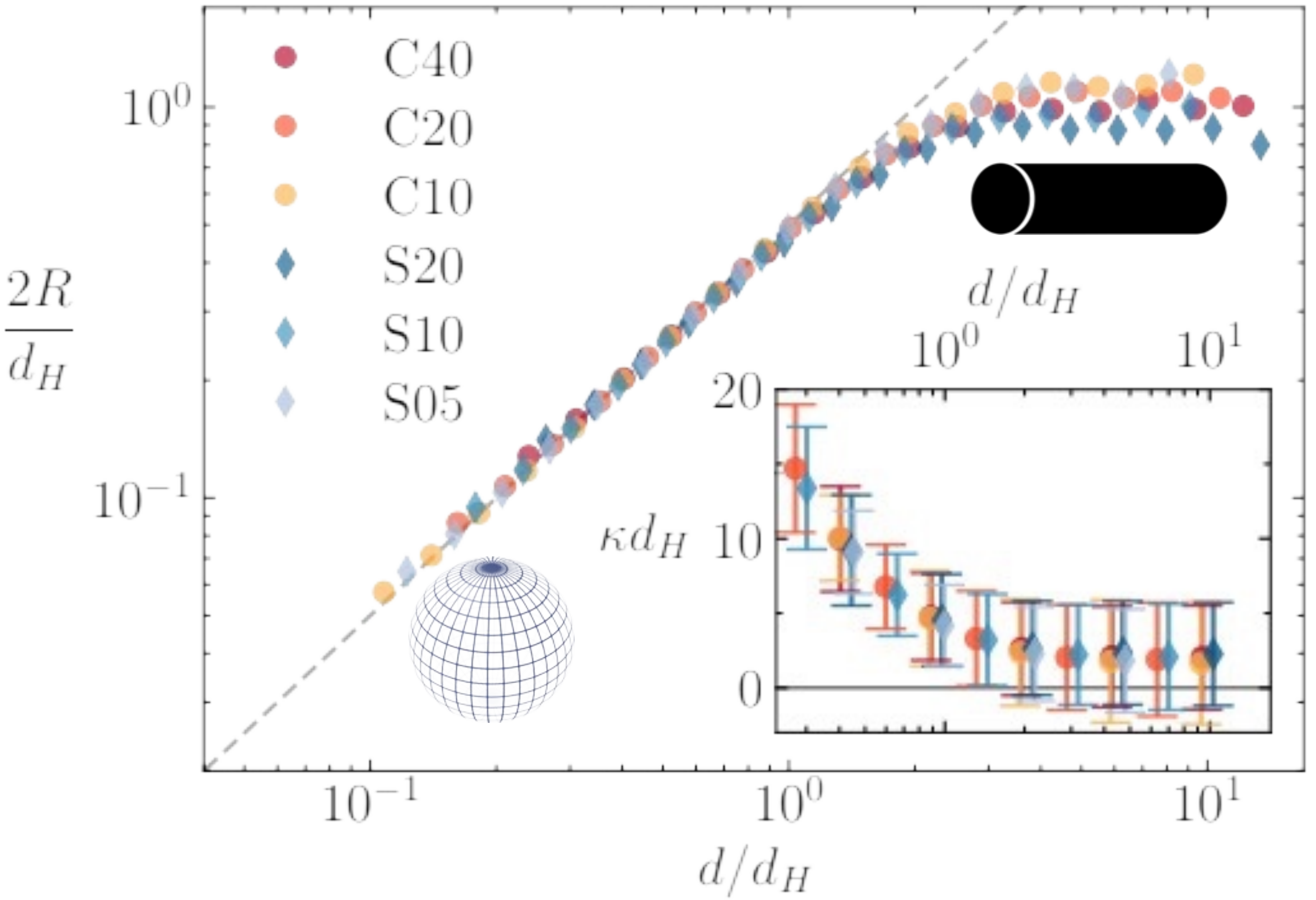
$$d = (6V/\pi)^{1/3}, \quad d_A = \sqrt{A/\pi}$$

- the aspect ratio decreases with droplet size, and rebounds in the breakup regime
- the sphericity decreases sharply in the breakup regime



Droplet radius of curvature

- We compute the curvature using the interface normal, $\kappa = \nabla \cdot \mathbf{n}$.
- The radius of curvature $R = 1/\kappa$.
- Below the Hinze-scale $R \approx d/4$
- Above the Hinze-scale $R \approx d_H/2$

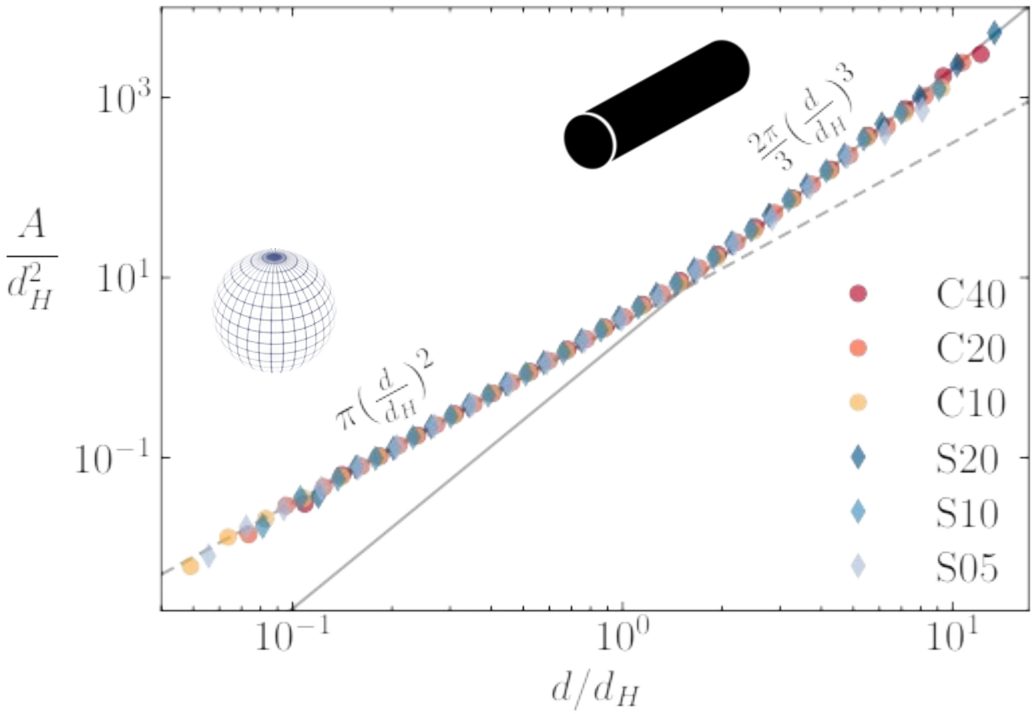


Droplet areas

- Below the Hinze-scale the droplet areas scale as spheres

- Above the Hinze-scale the droplet areas scale as filaments with diameter d_H

case		$\langle\phi\rangle$	$\langle\psi\rangle$	We
SP	●	0	0	-
C10	●	0.1	0	10
C20	●	0.1	0	20
C40	●	0.1	0	40
S05	♦	0.1	0.1	5
S10	♦	0.1	0.1	10
S20	♦	0.1	0.1	20

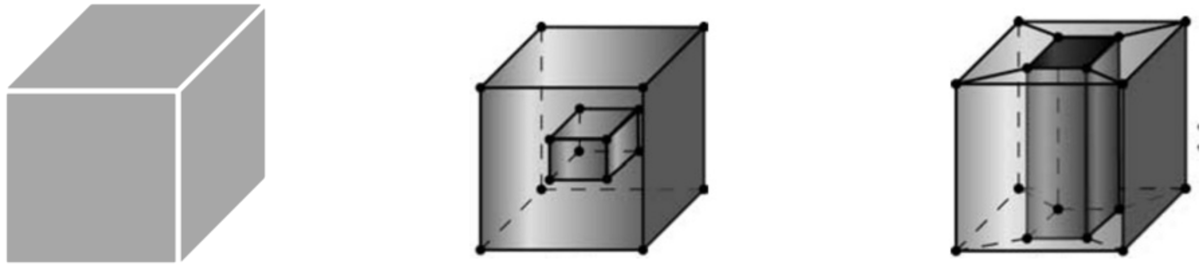


Topology of the droplets

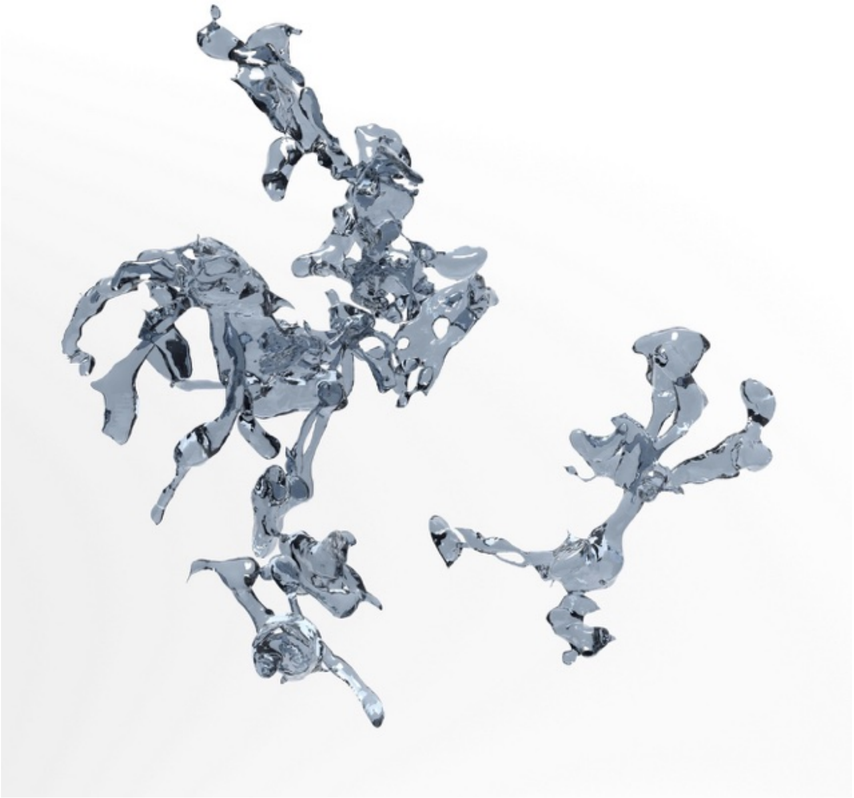
- We compute the Euler characteristic of the droplet interfaces $\chi = n - e + f.$
- This gives the number of handles h and voids v

$$1 - \chi/2 = h - v,$$

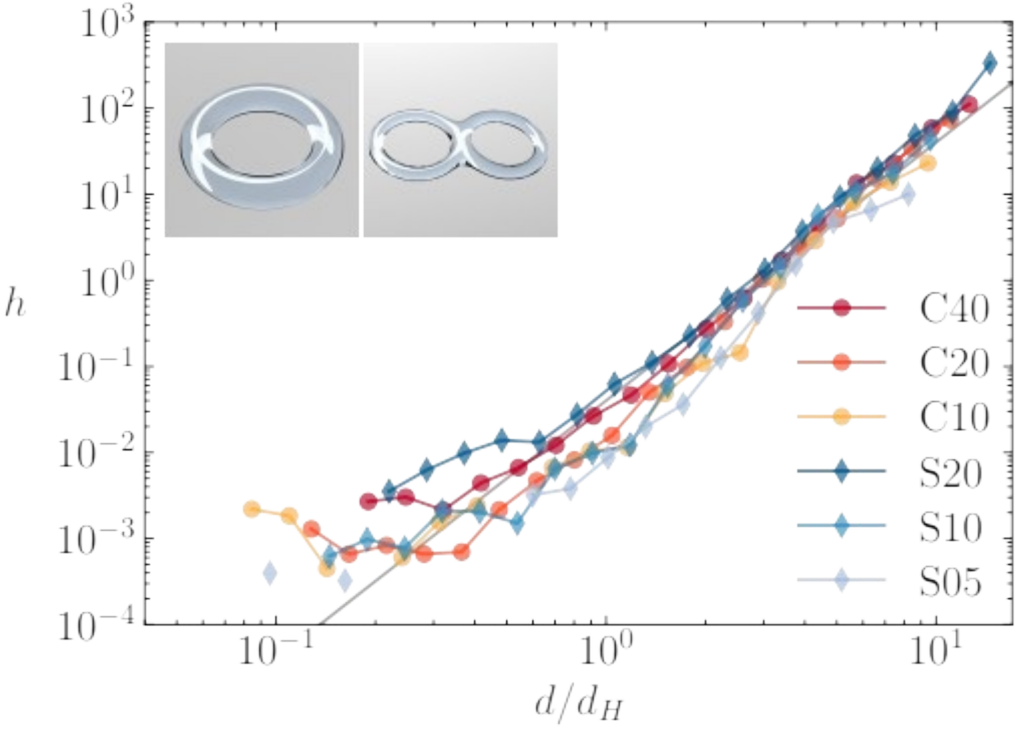
$h = 0, v = 0$	$h = 0, v = 1$	$h = 1, v = 0$

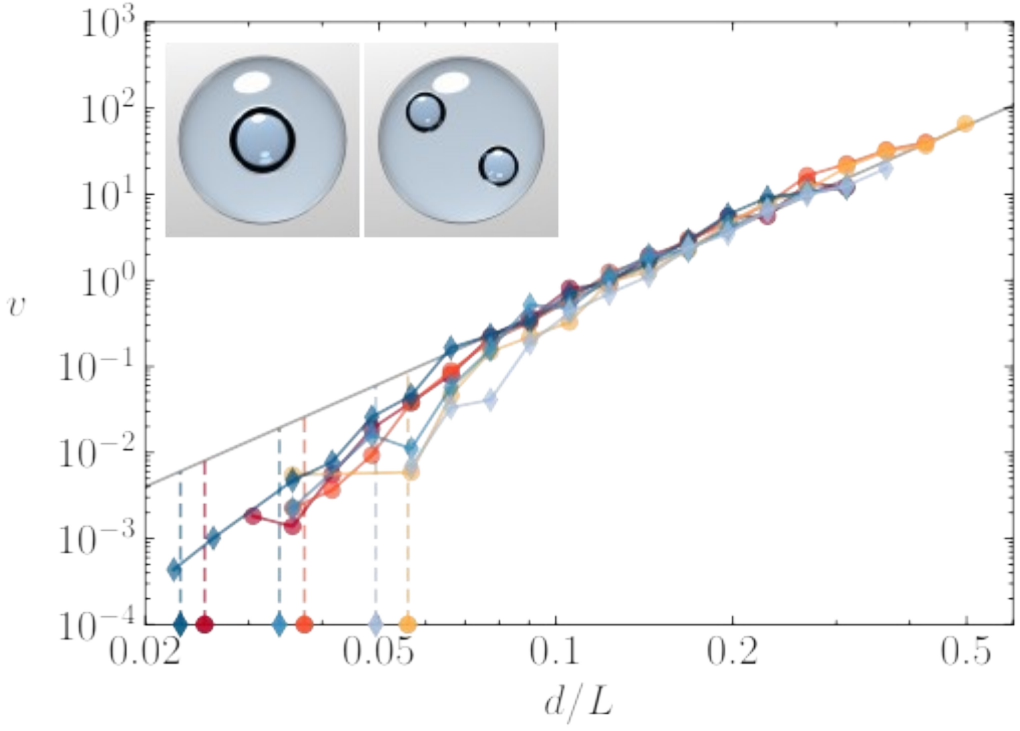


$n = 8$	$n = 16$	$n = 16$
$e = 12$	$e = 24$	$e = 32$
$f = 6$	$f = 12$	$f = 16$
$\chi = 2$	$\chi = 4$	$\chi = 0$



Handles and voids

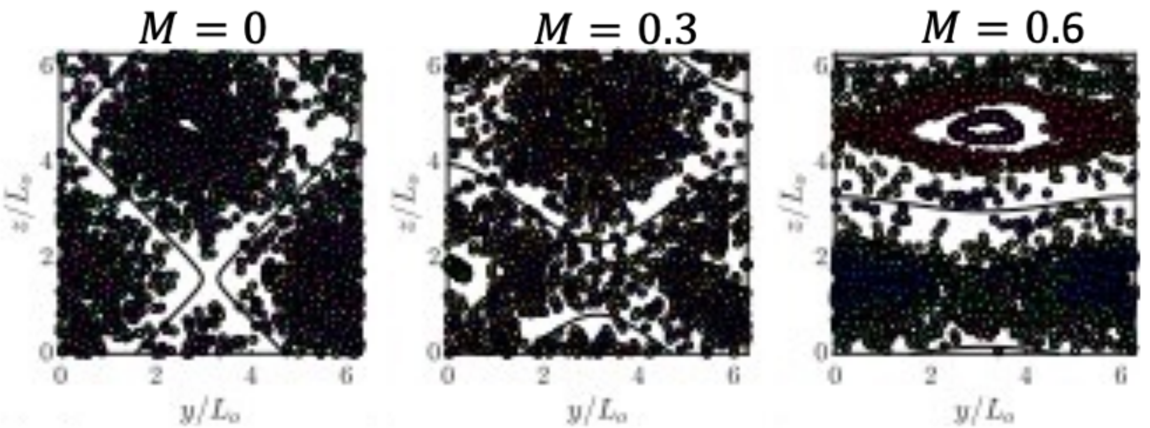




Conclusions

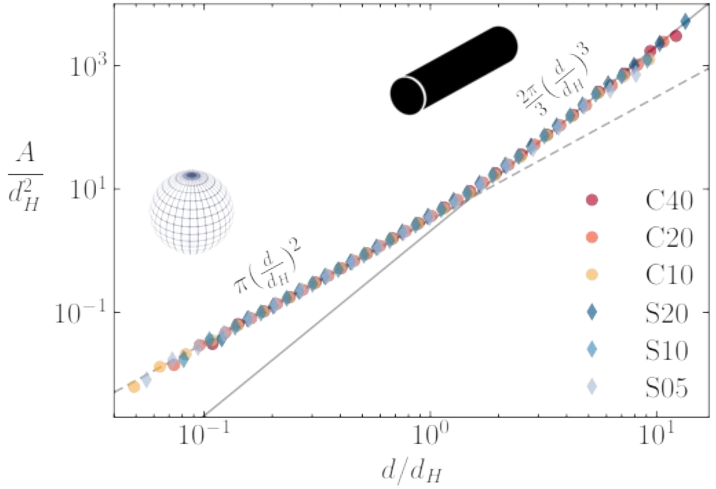
Part 1:

- The kinetic energy is enhanced at large particle mass fractions
- The flow becomes anisotropic
- The particles exhibit anomalous diffusion
- Chiarini, Cannon & Rosti, (2023). Anisotropic mean flow enhancement and anomalous transport of finite-size spherical particles in turbulent flows *(under review)*



Part 2:

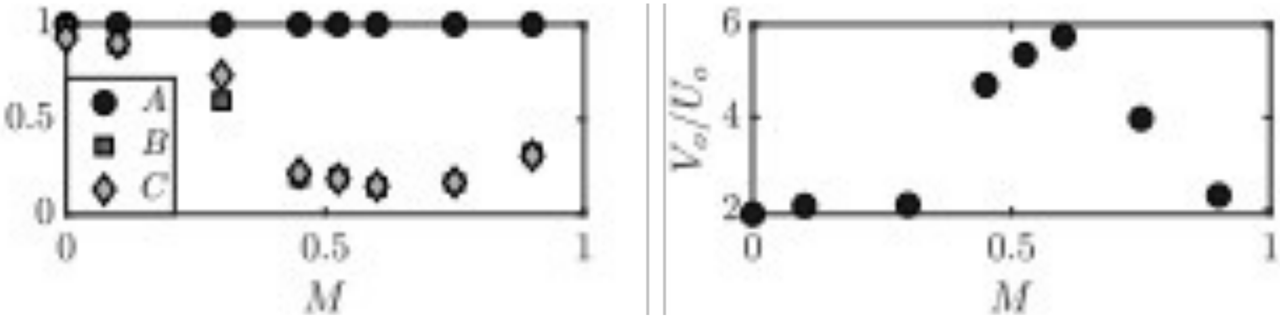
- The interface-averaged surface tension can be used to define the Hinze-scale in surfactant-laden flows
- Droplets smaller than the Hinze-scale are elipsoidal
- Droplets larger than the Hinze-scale are filamentous
- Cannon, Soligo & Rosti, (2023). Morphology of clean and surfactant-laden droplets in homogeneous isotropic turbulence *(under review) arxiv.org/abs/2307.15448*



Thank you!

Poincaré maps

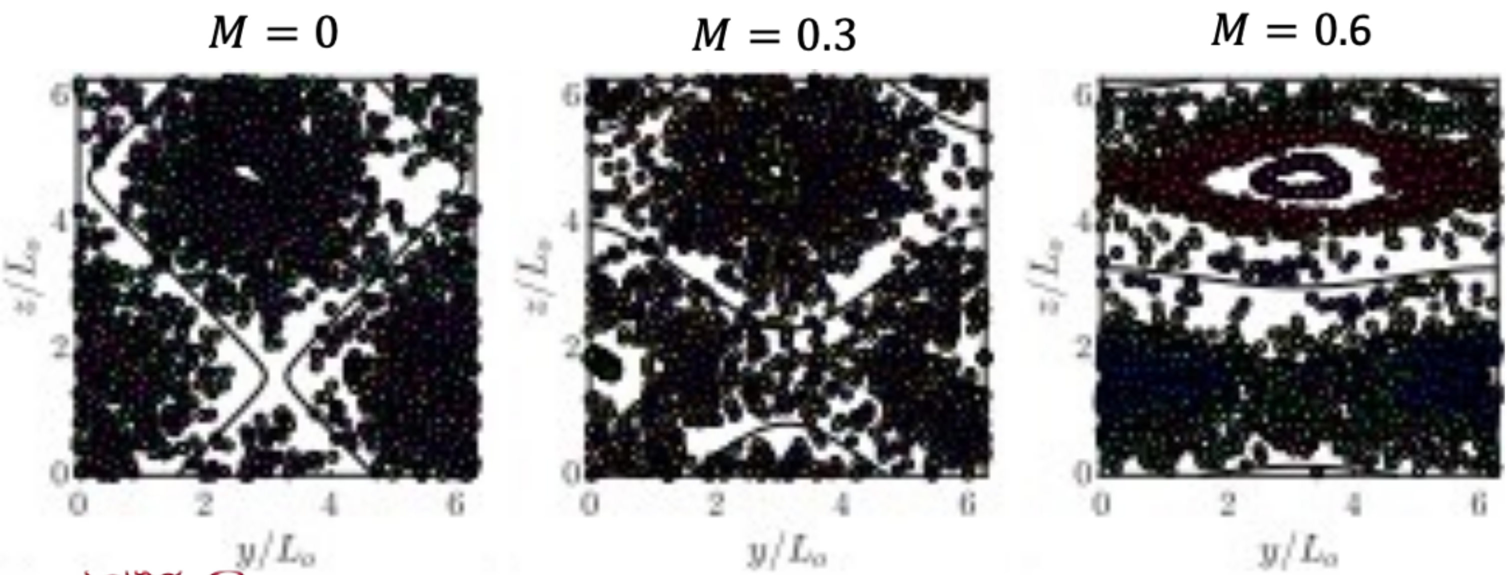
$$\boldsymbol{f} = \left(\frac{2\pi}{L}\right)^2 F_o \begin{pmatrix} A\sin(2\pi z/L) + C\sin(2\pi y/L) \\ B\sin(2\pi x/L) + A\sin(2\pi z/L) \\ C\sin(2\pi y/L) + B\sin(2\pi x/L) \end{pmatrix}$$



- Remarkably, the mean flow closely resembles the laminar ABC flow
- We can calculate the corresponding coefficients A,B and C with,

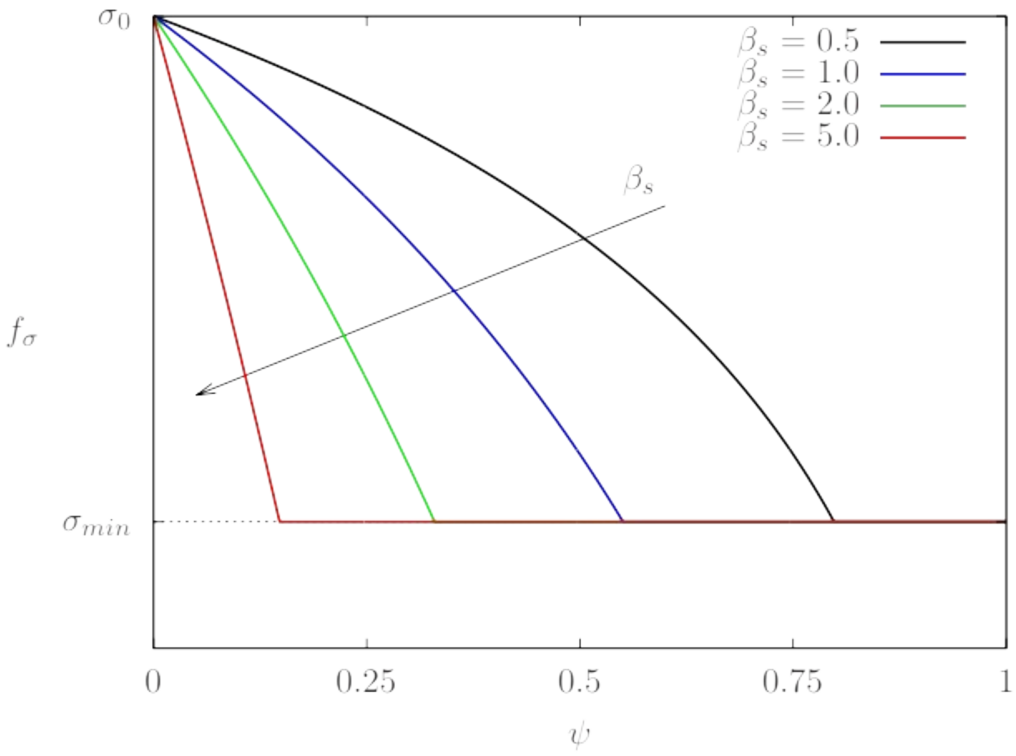
$$V_o A = \frac{1}{L^3}\int_0^L\int_0^L\int_0^L \left(U\sin(z/L_o) + V\cos(z/L_o)\right) dz\,dy\,dx$$

- For large mass fractions, the flow becomes anisotropic, and a state with $A \gg B \approx C$ is reached
- We show in black, the loci where the laminar ABC flow has $u = 0$.



Musacchio, *Phys. Rev. E* (2014)

Langmuir equation of state



$$f_\sigma = \max\left[\sigma_{min}, \sigma_0(1+\beta_s \log(1-\psi))\right]$$

Muradoglu, & Tryggvason, *J. Comput. Phys.* (2008)

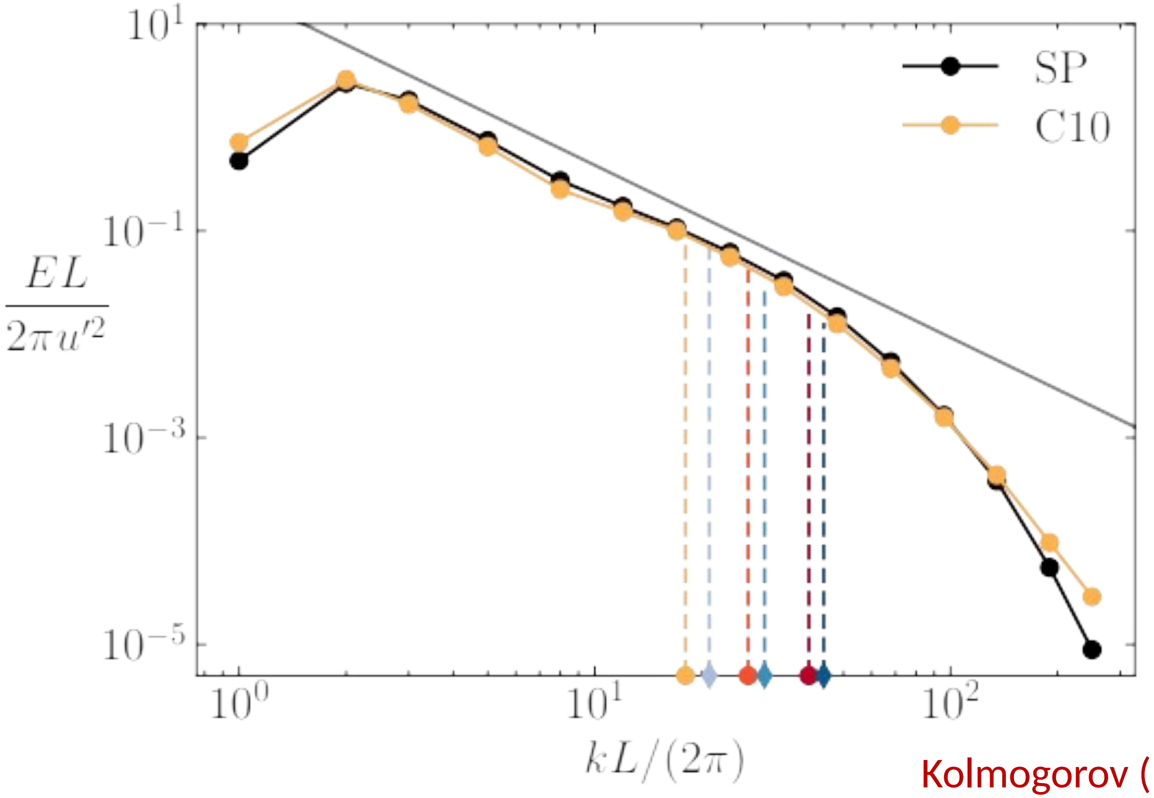
Kolmogorov-Hinze diameter

case	$\langle\phi\rangle$	$\langle\psi\rangle$	We
SP	0	0	-
C10	0.1	0	10
C20	0.1	0	20
C40	0.1	0	40
S05	0.1	0.1	5
S10	0.1	0.1	10
S20	0.1	0.1	20

- The turbulent kinetic energy spectrum has an $E{\sim}k^{-5/3}$ scaling
- The **Kolmogorov-Hinze diameter** is an estimate of the largest droplet which is not broken by turbulent fluctuations

$$d_H \equiv 0.725 \langle\sigma\rangle_I^{3/5} \rho^{-3/5} \varepsilon^{-2/5}$$

- The effective Weber number takes surfactants into account $We_e \equiv \rho u'^2 L_b / \langle\sigma\rangle_I$
- The total number of droplets increases with Weber number
- We define an equivalent diameter for each droplet, $d \equiv (6V/\pi)^{1/3}$.
- Droplet breakup and coalescence produce two distinct scalings in the DSD



Kolmogorov (1949)
Hinze, *AIChеJ* (1955)